

Unfolded Equations for Current Interactions of $4d$ Massless Fields as a Free System in Mixed Dimensions

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Abstract

Interactions of massless fields of all spins in four dimensions with currents of any spin is shown to result from a solution of the linear problem that describes a gluing between rank-one (massless) system and rank-two (current) system in the unfolded dynamics approach. Since the rank-two system is dual to a free rank-one higher-dimensional system, that effectively describes conformal fields in six space-time dimensions, the constructed system can be interpreted as describing a mixture between linear conformal fields in four and six dimensions. Interpretation of the obtained results in spirit of *AdS/CFT* correspondence is discussed.

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1 Introduction

We consider unfolded equations that describe current interactions of massless fields in four dimensional anti-de Sitter space. This work is based on the correspondence between fields and currents elaborated in [1] where the formulation of $4d$ massless fields in the ten-dimensional space \mathcal{M}_4 with matrix coordinates was studied. In [1] it was shown that \mathcal{M}_M admits different types of fields characterized by their rank, that satisfy different field equations. On the one hand it was shown that rank- r field equations are satisfied by products of r rank-one fields and it was observed that a rank-two field in \mathcal{M}_M generates conserved charges in \mathcal{M}_M . On the other hand, a rank- r field in \mathcal{M}_M was interpreted as a “compactification” of an “elementary” rank-one field in M_{rM} . Although the particular details are different, this correspondence is very much in spirit of the *AdC/CFT* correspondence [2, 3, 4] with a field in higher-dimensional (bulk) space-time identified with the current in a lower-dimensional (boundary) space-time. (It should be noted that this relation may involve duality transformations of field-source type.) We believe that this phenomenon has far going consequences, partially discussed already in [5], especially taking into account that the very notion of space-time dimension acquires dynamical origin in the framework of unfolded dynamics [6, 7].

Genuine massless fields in $d = 4$ are rank-one fields in \mathcal{M}_4 [5]. In [1, 8], it was shown that, for $M = 4$, the realization of a rank-two field in terms of bilinears of rank-one fields gives rise to the full list of conformal gauge invariant conserved currents of all spins in four dimensions [9], which generalize the so-called generalized Bell-Robinson currents constructed by Berends, Burgers and van Dam [10].

On the other hand the rank-two field in \mathcal{M}_4 can be identified with the elementary rank-one field in \mathcal{M}_8 that gives rise to usual conformal fields in six dimensions [11, 7, 12], which, in accordance with the general results of [13, 14], are the mixed symmetry fields described by various two-row rectangular Young diagrams. It should be noted that the idea that currents realized as bilinears of elementary fields behave as fields in higher dimension is not new and was discussed for example in [15, 16] (see also references therein). However, in the framework of higher-spin (HS) theories that describe infinite towers of massless fields of all spins this idea gets particularly neat realization.

This correspondence suggests the idea that the current interaction in four dimensions can be interpreted as a mixture between linear rank-one and rank-two fields in \mathcal{M}_4 , where the latter field is only assumed to satisfy the rank-two unfolded field equations. This implies that the seemingly nonlinear interaction of massless fields in four dimensions with the currents (that can be constructed from the same fields) results from a solution of the linear problem that describes a gluing between rank-one and rank-two fields in the unfolded dynamics approach. As mentioned above an interesting interpretation of this system is that it mixes massless fields in four space-time dimensions with conformal fields in six space-time dimensions interpreted as currents in the four-dimensional space.

The goal of this paper is to show how this works in practice. Namely, we present a linear unfolded system of equations that glues the unfolded equations of rank-one and rank-two fields in such a way that, upon realization of the rank-two fields in terms of bilinears of rank-

one fields, the usual field equations for massless fields receive corrections that just describe their current interactions. It is interesting to note that the same mechanism brings Yukawa interactions to the field equations of massless fields of spins 0 and 1/2.

The rest of the paper is organized as follows. In Section 2, we recall the unfolded form of $4d$ free HS field equations in AdS_4 proposed in [17, 18] and their flat limit. In Section 3, the constructions of conserved currents in the flat space of [1, 8] is recalled and its generalization to AdS_4 is given. The nontrivial current deformation of the rank-one unfolded system with the rank-two unfolded system is presented in Section 4. In Section 5, it is shown in detail how the deformed unfolded equations affect the form of dynamical equations for massless fields bringing currents to their right-hand sides. Section 6 contains summary of obtained results and discussion of further research directions. Appendices A, B and C collect technical details of the calculations.

2 Preliminaries

2.1 Higher spin gauge fields in AdS_4

In this Section we recall the unfolded form of $4d$ free HS field equations proposed in [17, 18]. It is based on the frame-like approach to HS gauge fields [19, 20] where a spin s HS gauge field is described by the set of 1-forms

$$\omega_{\alpha_1 \dots \alpha_k, \alpha'_1 \dots \alpha'_l} = dx^n \omega_{n \alpha_1 \dots \alpha_k, \alpha'_1 \dots \alpha'_l}, \quad k + l = 2(s - 1)$$

and the set of 0-forms $C_{\alpha_1 \dots \alpha_n, \beta'_1 \dots \beta'_m}(x)$ with $n - m = 2s$ along with their conjugates $\overline{C}_{\alpha_1 \dots \alpha_n, \beta'_1 \dots \beta'_m}(x)$ with $m - n = 2s$.¹ The HS gauge fields are self-conjugated $\overline{\omega_{\alpha_1 \dots \alpha_k, \beta'_1 \dots \beta'_l}} = \omega_{\beta_1 \dots \beta_l, \alpha'_1 \dots \alpha'_k}$. This set is equivalent to the real 1-form $\omega_{A_1 \dots A_{2(s-1)}}$ symmetric in the Majorana spinor indices $A = 1, \dots, 4$, that carries an irreducible module of the AdS_4 symmetry algebra $sp(4, \mathbb{R}) \sim o(3, 2)$.

AdS_4 is described by the Lorentz connection $w^{\alpha\beta}$, $\overline{w}^{\alpha'\beta'}$ and vierbein $e^{\alpha\alpha'}$. Altogether they form the $sp(4, \mathbb{R})$ connection $w^{AB} = w^{BA}$ that satisfies the $sp(4, \mathbb{R})$ zero curvature conditions

$$R^{AB} = 0, \quad R^{AB} = dw^{AB} + w^{AC} \wedge w_C^B, \quad (2.1)$$

where indices are raised and lowered by a $sp(4, \mathbb{R})$ invariant form $C_{AB} = -C_{BA}$

$$A_B = A^A C_{AB}, \quad A^A = C^{AB} A_B, \quad C_{AC} C^{BC} = \delta_A^B. \quad (2.2)$$

In terms of Lorentz components $w^{AB} = (w^{\alpha\beta}, \overline{w}^{\alpha'\beta'}, \lambda e^{\alpha\beta'}, \lambda e^{\beta\alpha'})$ where λ^{-1} is the AdS_4 radius, the AdS_4 equations (2.1) read as

$$R_{\alpha\beta} = 0, \quad \overline{R}_{\alpha'\beta'} = 0, \quad R_{\alpha\alpha'} = 0, \quad (2.3)$$

¹(Un)primed indices from the beginning of the Greek alphabet take two values $\alpha, \beta = 1, 2$ and $\alpha', \beta' = 1', 2'$. The two-component indices are raised and lowered as follows $A^\alpha = \varepsilon^{\alpha\beta} A_\beta$, $A_\alpha = \varepsilon_{\beta\alpha} A^\beta$, $\varepsilon_{\beta\alpha} = -\varepsilon_{\alpha\beta}$, $\varepsilon_{12} = 1$ and analogously for primed indices.

where

$$R_{\alpha\beta} = dw_{\alpha\beta} + w_{\alpha}{}^{\gamma} \wedge w_{\beta\gamma} + \lambda^2 e_{\alpha}{}^{\delta'} \wedge e_{\beta\delta'}, \quad (2.4)$$

$$\overline{R}_{\alpha'\beta'} = d\overline{w}_{\alpha'\beta'} + \overline{w}_{\alpha'}{}^{\gamma'} \wedge \overline{w}_{\beta'\gamma'} + \lambda^2 e^{\gamma}{}_{\alpha'} \wedge e_{\gamma\beta'},$$

$$R_{\alpha\beta'} = de_{\alpha\beta'} + w_{\alpha}{}^{\gamma} \wedge e_{\gamma\beta'} + \overline{w}_{\beta'}{}^{\delta'} \wedge e_{\alpha\delta'}. \quad (2.5)$$

The unfolded equations of motion of a spin- s massless field read as [18]

$$D^{ad}\omega(y, \bar{y}|x) = \overline{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \overline{C}(0, \bar{y} | x) + H^{\alpha\beta} \frac{\partial^2}{\partial y^{\alpha} \partial y^{\beta}} C(y, 0 | x), \quad (2.6)$$

$$D^{tw}C(y, \bar{y}|x) = 0, \quad (2.7)$$

where

$$H^{\alpha\beta} = e^{\alpha}{}_{\alpha'} \wedge e^{\beta\alpha'}, \quad \overline{H}^{\alpha'\beta'} = e_{\alpha}{}^{\alpha'} \wedge e^{\alpha\beta'}, \quad (2.8)$$

y^{α} and $\bar{y}^{\beta'}$ are auxiliary commuting conjugated two-component spinor variables, 1-form $\omega(y, \bar{y}|x)$ and 0-form $C(y, \bar{y}|x)$ have the form

$$\omega(y, \bar{y}|x) = \sum_{m,n \geq 0} \omega_{\alpha_1 \dots \alpha_n, \beta'_1 \dots \beta'_m}(x) \bar{y}^{\beta'_1} \dots \bar{y}^{\beta'_m},$$

with $n + m = 2(s - 1)$ (for $s \geq 1$),

$$C(y, \bar{y}|x) = \sum_{m,n \geq 0} C_{\alpha_1 \dots \alpha_n, \beta'_1 \dots \beta'_m}(x) y^{\alpha_1} \dots y^{\alpha_n} \dots \bar{y}^{\beta'_1} \dots \bar{y}^{\beta'_m},$$

with $n - m = 2s$, $\overline{C}(y, \bar{y}|x)$ is complex conjugated to $C(y, \bar{y}|x)$, and

$$D^{ad}\omega(y, \bar{y}|x) = D^L\omega(y, \bar{y}|x) - \lambda e^{\alpha\beta'} \left(y_{\alpha} \frac{\partial}{\partial \bar{y}^{\beta'}} + \frac{\partial}{\partial y^{\alpha}} \bar{y}_{\beta'} \right) \omega(y, \bar{y}|x), \quad (D^{ad})^2 = 0, \quad (2.9)$$

$$D^{tw}C(y, \bar{y}|x) = D^LC(y, \bar{y}|x) + \lambda e^{\alpha\beta'} \left(y_{\alpha} \bar{y}_{\beta'} + \frac{\partial^2}{\partial y^{\alpha} \partial \bar{y}^{\beta'}} \right) C(y, \bar{y}|x), \quad (D^{tw})^2 = 0, \quad (2.10)$$

where the Lorentz covariant derivative D^L is

$$D^LA(y, \bar{y}|x) = dA(y, \bar{y}|x) - \left(w^{\alpha\beta} y_{\alpha} \frac{\partial}{\partial y^{\beta}} + \overline{w}^{\alpha'\beta'} \bar{y}_{\alpha'} \frac{\partial}{\partial \bar{y}^{\beta'}} \right) A(y, \bar{y}|x). \quad (2.11)$$

$x^{\alpha\beta'} = x^n \sigma_n^{\alpha\beta'}$, are Minkowski coordinates where $\sigma_n^{\alpha\beta'}$ are four Hermitian 2×2 matrices.

As explained in [18, 21, 22], the dynamical massless fields are

- $C(x)$ and $\overline{C}(x)$ for two spin zero fields,
- $C_{\alpha}(x)$ and $\overline{C}_{\alpha'}(x)$ for a massless spin 1/2 field,
- $\omega_{\alpha_1 \dots \alpha_{s-1}, \alpha'_1 \dots \alpha'_{s-1}}(x)$ for an integer spin $s \geq 1$ massless field,

- $\omega_{\alpha_1 \dots \alpha_{s-3/2}, \alpha'_1 \dots \alpha'_{s-1/2}}(x)$ and its complex conjugate $\omega_{\alpha_1 \dots \alpha_{s-1/2}, \alpha'_1 \dots \alpha'_{s-3/2}}(x)$ for a half-integer spin $s \geq 3/2$ massless field.

All other fields are auxiliary, being expressed via derivatives of the dynamical massless fields by the equations (2.6) and (2.7).

The equations (2.7) are independent of (2.6) for spins $s = 0$ and $s = \frac{1}{2}$ and partially independent for spin one but become consequences of (2.6) for $s > 1$. The equations (2.6) express the holomorphic and antiholomorphic components of spin $s \geq 1$ 0-forms $C(y, \bar{y}|x)$ via derivatives of the massless field gauge 1-forms described by $\omega(y, \bar{y}|x)$. This identifies the spin $s \geq 1$ holomorphic and antiholomorphic components of the 0-forms $C(y, \bar{y}|x)$ with the Maxwell tensor, on-shell Rarita-Schwinger curvature, Weyl tensor and their HS generalizations. In addition, the equations (2.6) impose the standard field equations on the spin $s > 1$ massless gauge fields. The dynamical equations for spins $s \leq 1$ are contained in the equations (2.7).

2.2 σ_- -cohomology

In the unfolded dynamics approach, *dynamical fields*, their *differential gauge symmetries* (*i.e.*, those that are not Stueckelberg (*i.e.*, shift) symmetries) and *differential field equations* (*i.e.*, those that are not constraints) are characterized by the so-called σ_- -cohomology

Let us briefly recall the σ_- -cohomology analysis in Minkowski space following to [22]. A space V_0 , where 0-forms C, \bar{C} are valued, is endowed with the grading G_0

$$G_0 = \frac{1}{2}(n + \bar{n}), \quad n = y^\beta \frac{\partial}{\partial y^\beta}, \quad \bar{n} = \bar{y}^{\beta'} \frac{\partial}{\partial \bar{y}^{\beta'}}. \quad (2.12)$$

This gives

$$D^{tw} = D^L + \lambda \sigma_-^{tw} + \lambda \sigma_+^{tw}, \quad (2.13)$$

where

$$\sigma_-^{tw} = e^{\alpha\alpha'} \frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\alpha'}}, \quad \sigma_+^{tw} = e^{\alpha\alpha'} y_\alpha \bar{y}_{\alpha'}.$$

We have $[G_0, \sigma_\pm^{tw}] = \pm \sigma_\pm^{tw}$, $[G_0, D^L] = 0$, and $(\sigma_\pm^{tw})^2 = 0$.

A space V_1 , where 1-forms ω are valued, is endowed with the grading G_1

$$G_1 = \frac{1}{2} |n - \bar{n}|. \quad (2.14)$$

This gives

$$D^{ad} = D^L - \lambda \sigma_-^{ad} - \lambda \sigma_+^{ad}, \quad (2.15)$$

where

$$\sigma_-^{ad} = \rho_- \theta(n - \bar{n} - 2) + \overline{\rho_-} \theta(\bar{n} - n - 2), \quad \sigma_+^{ad} = \rho_- \theta(\bar{n} - n) + \overline{\rho_-} \theta(n - \bar{n}), \quad (2.16)$$

$$\rho_- = e^{\alpha\beta'} \frac{\partial}{\partial y^\alpha} \bar{y}_{\beta'}, \quad \overline{\rho_-} = e^{\alpha\beta'} \frac{\partial}{\partial \bar{y}^{\beta'}} y_\alpha, \quad \theta(m) = 1 \ (0), \quad m \geq 0 \ (m < 0). \quad (2.17)$$

We have $[G_1, \sigma_{\pm}^{ad}] = \pm \sigma_{\pm}^{ad}$, $[G_1, \mathcal{D}^L] = 0$. Although ρ_- and $\bar{\rho}_-$ do not anticommute, $(\sigma_-^{ad})^2 = 0$ because $(\rho_-)^2 = (\bar{\rho}_-)^2 = 0$ and the step functions guarantee that the parts of σ_- associated with ρ_- and $\bar{\rho}_-$ act in different spaces.

Setting

$$\sigma_- = \sigma_-^{tw} + \sigma_-^{ad}, \quad (2.18)$$

where σ_-^{tw} acts on zero-forms, while σ_-^{ad} acts on one-forms, cohomology of σ_- determines the dynamical content of the dynamical system at hand. Namely, from the level-by-level analysis of the equations (2.6) and (2.7) it follows that all fields that do not belong to $\text{Ker } \sigma_-$ are auxiliary, being expressed by (2.6) and (2.7) via derivatives of the lower grade fields. (For more detail see e.g. [23, 22].) In the case of massless fields, the nontrivial cohomology of σ_- is concentrated in the subspaces with $G_j = 0$ and $\pm 1/2$ [22]. In particular, the nontrivial cohomology of $H^0(\sigma_-)$ appears in the subspaces of grades $G_1 = 0$ or $1/2$, where σ_- acts trivially because of the step functions in (2.16).

2.3 Flat limit

To take the flat limit it is necessary to perform certain rescalings. To this end, it is useful to introduce notations [22] A_{\pm} and A_0 so that the spectrum of the operator $\left(y^{\alpha} \frac{\partial}{\partial y^{\alpha}} - \bar{y}^{\alpha'} \frac{\partial}{\partial \bar{y}^{\alpha'}}\right)$ is positive on $A_+(y, \bar{y} | x)$, negative on $A_-(y, \bar{y} | x)$ and zero on $A_0(y, \bar{y} | x)$. Having the decomposition

$$A(y, \bar{y} | x) = A_+(y, \bar{y} | x) + A_-(y, \bar{y} | x) + A_0(y, \bar{y} | x), \quad (2.19)$$

the rescaled field is introduced as follows

$$\tilde{A}(y, \bar{y} | x) = A_+(\lambda y, \bar{y} | x) + A_-(y, \lambda \bar{y} | x) + A_0(\lambda y, \bar{y} | x). \quad (2.20)$$

(Note that $A_0(\lambda y, \bar{y} | x) = A_0(y, \lambda \bar{y} | x)$). For the rescaled variables, the flat limit $\lambda \rightarrow 0$ of the adjoint and twisted adjoint covariant derivatives (2.9) and (2.10) gives

$$D_{fl}^{ad} \tilde{A}(y, \bar{y} | x) = D^L \tilde{A}(y, \bar{y} | x) - e^{\alpha\beta'} \left(y_{\alpha} \frac{\partial}{\partial \bar{y}^{\beta'}} \tilde{A}_-(y, \bar{y} | x) + \frac{\partial}{\partial y^{\alpha}} \bar{y}_{\beta'} \tilde{A}_+(y, \bar{y} | x) \right), \quad (2.21)$$

$$D_{fl}^{tw} \tilde{A}(y, \bar{y} | x) = D^L \tilde{A}(y, \bar{y} | x) + e^{\alpha\beta'} \frac{\partial^2}{\partial y^{\alpha} \partial \bar{y}^{\beta'}} \tilde{A}(y, \bar{y} | x). \quad (2.22)$$

The flat limit of the unfolded massless equations results from (2.6) and (2.7) via the substitution of D^L and e of Minkowski space and the replacement of D^{ad} and D^{tw} by D_{fl}^{ad} and D_{fl}^{tw} , respectively. The resulting field equations describe free HS fields in Minkowski space. Let us stress that the flat limit prescription (2.20), that may look somewhat unnatural in the two-component spinor notation, is designed just to give rise to the theory of Fronsdal [24] and Fang and Fronsdal [25] (for more detail see [22]).

Note that, although the contraction $\lambda \rightarrow 0$ with the rescaling (2.20) is consistent with the free HS field equations, it turns out to be inconsistent in the nonlinear HS theory because negative powers of λ survive in the full nonlinear equations upon the rescaling (2.20), not allowing the flat limit in the nonlinear theory. This is why the Minkowski background is unreachable in the non-linear HS gauge theories of [26, 27, 28].

2.4 Unfolded equations in matrix spaces \mathcal{M}_M

As observed in [5], the massless equations (2.7) can be promoted to a larger space \mathcal{M}_4 with matrix coordinates $X^{AB} = X^{BA}$ by extending the system (2.7) to

$$dX^{AB} \left(\frac{\partial}{\partial X^{AB}} \pm \frac{\partial^2}{\partial Y^A \partial Y^B} \right) C_{\pm}(Y|X) = 0, \quad (2.23)$$

where the \pm sign is introduced for future convenience. This extension makes the $Sp(8)$ symmetry of the tower of massless fields of all spins, observed originally by Fronsdal [29], geometrically realized on the Lagrangian Grassmannian that was shown in [29] to be a minimal $Sp(8)$ invariant space that contains Minkowski space as a subspace. (Note that in [11] it was also observed that the tower of $4d$ massless fields of all spins is naturally realized in \mathcal{M}_4 .)

The system (2.23) extends to \mathcal{M}_4 the $4d$ massless equations in Minkowski background formulated in Cartesian coordinates. Its extension to a AdS -like version of \mathcal{M}_4 , which is the group manifold $Sp(4)$ [5], is also available [30] in any coordinate system. Note that more recently the one-form sector of HS equations (2.6) was also extended to \mathcal{M}_4 in [22].

According to general properties of unfolded equations, the equations (2.23) are equivalent to the flat limit of the original $4d$ HS equations (2.7). The interesting details of this correspondence were worked out in [7, 12].

In [1], the construction of equations (2.23) was extended to so-called rank- r systems of the form

$$dX^{AB} \left(\frac{\partial}{\partial X^{AB}} \pm \eta^{ij} \frac{\partial^2}{\partial Y^{iA} \partial Y^{jB}} \right) C_{\pm}^r(Y|X) = 0, \quad (2.24)$$

where $i, j = 1 \dots r$ and $\eta^{ij} = \eta^{ji}$ is some nondegenerate metric. The following two comments on the properties of higher rank systems are most relevant to the analysis of this paper.

One is that in the basis where η^{ij} is diagonal, the higher-rank equations (2.24) are satisfied by the products of rank-one fields

$$C^r(Y_i|X) = C_1(Y_1|X) C_2(Y_2|X) \dots C_r(Y_r|X). \quad (2.25)$$

Another one is that the rank- r systems in \mathcal{M}_M can further be extended to a rank-one system (2.23) in the larger space \mathcal{M}_{rM} . However, as was shown in [11, 7, 12] the rank- r fields in \mathcal{M}_M with higher M describe conformal fields in diverse space-time dimensions. In particular, a rank-one field in \mathcal{M}_8 describes all conformal fields in the six-dimensional Minkowski space. This implies that conformal currents in four space-time dimensions, that were shown in [8] to be described by rank-two fields in \mathcal{M}_4 , are equivalent to conformal fields in six space-time dimensions. More precisely we should say that the $4d$ currents are dual to the $6d$ conformal fields. The reason is that fields are represented by the product of C_- fields in (2.23) while the currents are represented by the product of C_+ and C_- , where C_+ and C_- describe, respectively, particles and anti-particles, *i.e.*, the space of single-particle states and its dual.² In this paper we will loosely identify the currents with the fields.

²Strictly speaking this interpretation requires an additional factor of i in the second term of (2.23), skipped in this paper. For more detail in these issues we refer the reader to [8].

Now we are in a position to explain how rank-two equations give rise to conserved currents considering for simplicity the reduction of \mathcal{M}_4 to the usual Minkowski space.

3 Conserved currents

3.1 Minkowski case

The reduction to Minkowski space of the rank-two field equations of [8] reads as

$$D_{fl2}{}^{tw} J(y^\pm, \bar{y}^\pm | x) = \left(D^L + \hbar e^{\alpha\beta'} \left(\frac{\partial^2}{\partial y^{+\alpha} \partial \bar{y}^{-\beta'}} + \frac{\partial^2}{\partial y^{-\alpha} \partial \bar{y}^{+\beta'}} \right) \right) J(y^\pm, \bar{y}^\pm | x) = 0 \quad (3.1)$$

for any parameter \hbar introduced for the future convenience. It is easy to check that the 3-form

$$\Omega(J) = \mathcal{H}^{\alpha\alpha'} \frac{\partial}{\partial y^{-\alpha}} \frac{\partial}{\partial \bar{y}^{-\alpha'}} J(y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0}, \quad (3.2)$$

is closed provided that $J(y^\pm, \bar{y}^\pm | x)$ satisfies Eq. (3.1) and

$$\mathcal{H}^{\alpha\delta'} = -\frac{1}{3} e^\alpha{}_{\alpha'} \wedge e^{\beta\alpha'} \wedge e_\beta{}^{\delta'}. \quad (3.3)$$

To define symmetry parameters that produce new conserved currents, consider the *adjoint* covariant derivative

$$D_{fl2}{}^{ad} = D^L + \hbar e^{\alpha\beta'} \left(u_{-\alpha} \frac{\partial}{\partial \bar{y}^{+\beta'}} + \bar{u}_{-\beta'} \frac{\partial}{\partial y^{+\alpha}} \right), \quad (3.4)$$

that can be obtained from $D_{fl2}{}^{tw}$ by the substitution, that formally coincides with the “half Fourier transform” of [8],

$$y^{-\alpha} \rightarrow -\frac{\partial}{\partial u_{-\alpha}}, \quad \bar{y}^{-\alpha'} \rightarrow -\frac{\partial}{\partial \bar{u}_{-\alpha'}}, \quad \frac{\partial}{\partial y^{-\alpha}} \rightarrow u_{-\alpha}, \quad \frac{\partial}{\partial \bar{y}^{-\alpha'}} \rightarrow \bar{u}_{-\alpha'}. \quad (3.5)$$

Since the covariant derivative (3.4) is of first order, the space of regular solutions of the equation

$$D'_{fl2}{}^{tw} \xi(y^+, \bar{y}^+, u_-, \bar{u}_- | x) = 0 \quad (3.6)$$

forms a commutative algebra \mathcal{R}'_{fl} . Evidently, \mathcal{R}'_{fl} is generated by the elementary solutions

$$\xi'_\beta = u_{-\beta}, \quad \xi'^{+\alpha} = y^{+\alpha} - \hbar x^{\alpha\beta'} \bar{u}_{-\beta'}, \quad \bar{\xi}'_{\beta'} = \bar{u}_{-\beta'}, \quad \bar{\xi}'^{+\alpha'} = \bar{y}^{+\alpha'} - \hbar x^{\beta\alpha'} u_{-\beta}. \quad (3.7)$$

By the inverse to (3.5) substitution

$$u_{-\alpha} \rightarrow \frac{\partial}{\partial y^{-\alpha}}, \quad \bar{u}_{-\alpha'} \rightarrow \frac{\partial}{\partial \bar{y}^{-\alpha'}}, \quad \frac{\partial}{\partial u_{-\alpha}} \rightarrow -y^{-\alpha}, \quad \frac{\partial}{\partial \bar{u}_{-\alpha'}} \rightarrow -\bar{y}^{-\alpha'} \quad (3.8)$$

the algebra \mathcal{R}'_{fl} is mapped to the algebra \mathcal{R}_{fl} of differential operators $\eta(\xi'_{-\beta}, \bar{\xi}'_{-\beta'}, \xi'^{+\alpha}, \bar{\xi}'^{+\alpha'})$ generated by the images of operators ξ' and $\bar{\xi}'$ (3.7)

$$\xi'_{-\alpha} = \frac{\partial}{\partial y^{-\alpha}}, \quad \bar{\xi}'_{-\beta'} = \frac{\partial}{\partial \bar{y}^{-\beta'}}, \quad \xi'^{+\alpha} = y^{+\alpha} - x^{\alpha\beta'} \frac{\partial}{\partial \bar{y}^{-\beta'}}, \quad \bar{\xi}'^{+\alpha'} = \bar{y}^{+\alpha'} - x^{\beta\alpha'} \frac{\partial}{\partial y^{-\beta}}. \quad (3.9)$$

Since the parameter $\eta(\xi'_{-\beta}, \bar{\xi}'_{-\beta'}, \xi'^{+\alpha}, \bar{\xi}'^{+\alpha'}) \in \mathcal{R}_{fl}$ satisfies

$$[D_{fl2}{}^{tw}, \eta] = 0,$$

it follows

$$D_{fl2}{}^{tw} J(y^{\pm}, \bar{y}^{\pm}|x) = 0 \quad \Rightarrow \quad D_{fl2}{}^{tw} \eta(J(y^{\pm}, \bar{y}^{\pm}|x)) = 0.$$

Since any element of \mathcal{R}_{fl} generates a conservation law, as explained in more detail in [8], \mathcal{R}_{fl} is the space of HS global symmetry parameters. Indeed, it matches the space of HS global symmetry parameters of [9]. Note also that the set of generators (3.9) is just the set of 4d conformal supergenerators, that act on solutions of the unfolded system (2.7).

The relation with usual currents is due to the fact that for any parameter $\eta \in \mathcal{R}_{fl}$ Eq. (3.1) is solved by the bilinear [1]

$$J(y^{\pm} \bar{y}^{\pm}|x) = \eta C_+(y^+ + y^-, \bar{y}^+ + \bar{y}^-|x) C_-(y^+ - y^-, \bar{y}^+ - \bar{y}^-|x), \quad (3.10)$$

of rank one fields $C_{\pm}(y \bar{y}|x)$ that solve the rank one equations (3.1) with $\hbar = \pm 1$. The resulting currents reproduce various expressions for the lower-spin and HS conserved currents built of massless fields, originally obtained in [10].

3.2 AdS_4

In the case of AdS_4 the rank-two unfolded equations are

$$D_2{}^{tw} J(y^{\pm}, \bar{y}^{\pm}|x) = 0, \quad (3.11)$$

where

$$D_2{}^{tw} = D^L + \lambda e^{\alpha\beta'} \left(y^+_{\alpha} \bar{y}^{-\beta'} + y^{-\gamma} \bar{y}^{+\beta'} + \frac{\partial^2}{\partial y^{+\alpha} \partial \bar{y}^{-\beta'}} + \frac{\partial^2}{\partial y^{-\alpha} \partial \bar{y}^{+\beta'}} \right). \quad (3.12)$$

Again, they imply that the differential 3-form (3.2) is closed, which is easy to see using that $y^{\pm} = \bar{y}^{\pm} = 0$.

Proceeding as in the previous subsection, to find symmetry parameters of AdS_4 currents we have to solve the equation

$$D_2{}^{ad} \xi(y^+, \bar{y}^+, u_-, \bar{u}_-|x) = 0, \quad (3.13)$$

$$D_2{}^{ad} = D^L + \lambda e^{\alpha\beta'} \left(-y_{+\alpha} \frac{\partial}{\partial \bar{u}^{-\beta'}} - \bar{y}_{+\beta'} \frac{\partial}{\partial u^{-\alpha}} + u_{-\alpha} \frac{\partial}{\partial \bar{y}^{+\beta'}} + \bar{u}_{-\beta'} \frac{\partial}{\partial y^{+\alpha}} \right),$$

using that in the AdS_4 case D_2^{ad} is also obtained from D_2^{tw} by the substitution (3.5).

Since Eq. (3.13) is again a first order system of partial differential equations, the space of its solutions forms a commutative algebra. Since the compatibility of the equation (3.13) is guaranteed by the flatness condition (2.3), the space of its solutions is isomorphic to the space of arbitrary functions of $y^+, \bar{y}^+, u_-, \bar{u}_-$ i.e., $\xi(y^+, \bar{y}^+, u_-, \bar{u}_-|x)$ is reconstructed via its values at any given point $x = x_0$. Since the equation (3.13) is homogeneous in the variables $y^+, \bar{y}^+, u_-, \bar{u}_-$ its solutions can also be chosen to be homogeneous. Moreover, since D_2^{ad} commutes to grading operators $y^{+\alpha} \frac{\partial}{\partial y^{+\alpha}} + \bar{u}^{-\alpha'} \frac{\partial}{\partial \bar{u}^{-\alpha'}}$ and $u^{-\alpha} \frac{\partial}{\partial y^{-\alpha}} + \bar{y}^{+\alpha'} \frac{\partial}{\partial \bar{y}^{+\alpha'}}$, to describe a general solution of the equation (3.13) it is enough to find a full set of solutions linear either in y^+ and \bar{u}_- or in u_- and \bar{y}^+ .

Let $\varrho_\alpha(u_-, \bar{y}^+|x)$, $\epsilon^\alpha(y^+, \bar{u}_-|x)$, $\bar{\varrho}_{\alpha'}(u_-, \bar{y}^+|x)$ and $\bar{\epsilon}^{\alpha'}(y^+, \bar{u}_-|x)$ denote those solutions, that satisfy the conditions

$$\varrho_\alpha(u_-, 0|0) = u_{-\alpha}, \quad \epsilon^\alpha(y^+, 0|0) = y^{+\alpha}, \quad \bar{\epsilon}^{\alpha'}(0, \bar{y}^+|0) = \bar{y}^{+\alpha'}, \quad \bar{\varrho}_{\alpha'}(0, \bar{u}_-|0) = \bar{u}_{-\alpha'}.$$

These are simply the parameters of global conformal SUSY in AdS_4 . The general HS parameter is an arbitrary polynomial of the supersymmetry parameters

$$\eta'(y^+, \bar{y}^+, u_-, \bar{u}_-|x) = P(\varrho_\alpha, \epsilon^\alpha, \bar{\varrho}_{\alpha'}, \bar{\epsilon}^{\alpha'}). \quad (3.14)$$

As in Minkowski case, the inverse substitution (3.8) maps the algebra \mathcal{R}'_{AdS} to the algebra \mathcal{R}_{AdS} of differential operators generated by $\varrho_\alpha(\frac{\partial}{\partial y^-}, \bar{y}^+|x)$, $\epsilon^\alpha(y^+, \frac{\partial}{\partial \bar{y}^-}|x)$, $\bar{\varrho}_{\alpha'}(\frac{\partial}{\partial y^-}, \bar{y}^+|x)$ and $\bar{\epsilon}^{\alpha'}(y^+, \frac{\partial}{\partial \bar{y}^-}|x)$. Again it follows that

$$D'^{tw}_\eta J(y^\pm, \bar{y}^\pm|x) = 0$$

provided that $\eta \in \mathcal{R}_{AdS}$ and $J(y^\pm, \bar{y}^\pm|x)$ satisfies (3.11).

To introduce currents, that are bilinear in rank-one fields, it is convenient to consider the operators D^{tw}_\pm , that differ from D^{tw} (2.10) by a sign in front of λ so that the corresponding rank-one field equations are

$$D^{tw}_\pm \mathcal{C}_\pm(y, \bar{y}|x) = D^L \mathcal{C}_\pm(y, \bar{y}|x) \pm \lambda e^{\alpha\beta'} \left(y_\alpha \bar{y}_{\beta'} + \frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\beta'}} \right) \mathcal{C}_\pm(y, \bar{y}|x). \quad (3.15)$$

Analogously to the flat limit case, for any parameter $\eta \in \mathcal{R}_{AdS}$, Eq. (3.11) is solved by the bilinear

$$J(y^\pm, \bar{y}^\pm|x) = \eta \mathcal{C}_+(y^+ + y^-, \bar{y}^+ + \bar{y}^-|x) \mathcal{C}_-(y^+ - y^-, \bar{y}^+ - \bar{y}^-|x) \quad (3.16)$$

of rank-one fields $\mathcal{C}_\pm(\sqrt{2}y, \sqrt{2}\bar{y}|x)$ that solve the rank-one equations (3.15).

3.3 Howe dual algebra

To sort out different solutions of the rank-two equation (3.11), we observe that the operators

$$\begin{aligned} f_- &= \frac{\partial}{\partial y^{+\gamma}} \frac{\partial}{\partial y^{-\gamma}} - \bar{y}^{+\gamma'} \bar{y}^{-\gamma'} , & f_+ &= y^{+\nu} y^{-\nu} - \frac{\partial}{\partial \bar{y}^{+\nu'}} \frac{\partial}{\partial \bar{y}^{-\nu'}} , \\ h &= y^{+\alpha} \frac{\partial}{\partial y^{+\alpha}} - \bar{y}^{-\alpha'} \frac{\partial}{\partial \bar{y}^{-\alpha'}} + y^{-\alpha} \frac{\partial}{\partial y^{-\alpha}} - \bar{y}^{+\alpha'} \frac{\partial}{\partial \bar{y}^{+\alpha'}} , \end{aligned} \quad (3.17)$$

and

$$v = y^{+\alpha} \frac{\partial}{\partial y^{+\alpha}} - \bar{y}^{+\alpha'} \frac{\partial}{\partial \bar{y}^{+\alpha'}} - y^{-\alpha} \frac{\partial}{\partial y^{-\alpha}} + \bar{y}^{-\alpha'} \frac{\partial}{\partial \bar{y}^{-\alpha'}} \quad (3.18)$$

commute to D_2^{tw} . These operators form a Lie algebra gl_2 with nonzero (sl_2) commutation relations

$$[f_-, f_+] = h, \quad [h, f_+] = 2f_+, \quad [h, f_-] = -2f_-.$$

The operator v is central.

Clearly, the system of equations (3.11) decomposes into a set of subsystems characterized by different elements of gl_2 -modules realized on the solutions of the equation (3.11).

Let

$$Y = y^{+\nu} y^{-\nu}, \quad \bar{Y} = \bar{y}^{+\nu'} \bar{y}^{-\nu'}. \quad (3.19)$$

Any polynomial of y^\pm, \bar{y}^\pm can be represented as a sum of polynomials

$$\bar{Y}^l Y^n C_{\alpha(m+k)} y^{+\alpha(m)} y^{-\alpha(k)},$$

where $C_{\alpha(m+k)}$ is a symmetric multispinor and $l, n, k, m \in \mathbb{N}$. It is easy to see that

$$\frac{\partial}{\partial \bar{y}^{+\nu'}} \frac{\partial}{\partial \bar{y}^{-\nu'}} Y^n C_{\alpha(m+k)} y^{+\alpha(m)} y^{-\alpha(k)} = -n(n+1+m+k) Y^{n-1} C_{\alpha(m+k)} y^{+\alpha(m)} y^{-\alpha(k)}.$$

It follows that, for any order m homogeneous polynomial $s^m(y^\pm)$ and function $f(\bar{y}^\pm, \bar{Y})$, that satisfy

$$\frac{\partial^2}{\partial y_\alpha^- \partial y^{+\alpha}} s^m(y^\pm) = 0, \quad \frac{\partial^2}{\partial y_\alpha^- \partial y^{+\alpha}} f(\bar{y}^\pm, \bar{Y}) = 0, \quad (3.20)$$

$$F(y, \bar{y}, Y, \bar{Y}) = f(\bar{y}^\pm, \bar{Y}) s^m(y^\pm) \sum_n Y^n \bar{Y}^n \frac{(-1)^n}{(1+m+n)!n!} \quad (3.21)$$

is a lowest vector of sl_2 . Note that $F(y, \bar{y}, Y, \bar{Y})$ satisfies

$$\left(\left(Y \frac{\partial}{\partial Y} + y^{A\alpha} \frac{\partial}{\partial y^{A\alpha}} + 1 \right) \frac{\partial}{\partial Y} + \bar{Y} \right) F(y, \bar{y}, Y, \bar{Y}) = 0.$$

Therefore a singlet of sl_2 has the form

$$F_0(y, \bar{y}, Y, \bar{Y}) = S^m(y^\pm, \bar{y}^\pm) \sum_n (y^{+\nu} y^{-\nu})^n (\bar{y}^{+\nu'} \bar{y}^{-\nu'})^n \frac{(-1)^n}{(1+m+n)!n!}, \quad (3.22)$$

where $S^m(y^\pm, \bar{y}^\pm)$ is any polynomial obeying the properties

$$\left[\frac{\partial}{\partial y^{+\gamma}} \frac{\partial}{\partial y^{-\gamma}}, S^m(y^\pm, \bar{y}^\pm) \right] = 0, \quad \left[y^{A\alpha} \frac{\partial}{\partial y^{A\alpha}}, S^m(y^\pm, \bar{y}^\pm) \right] = m S^m(y^\pm, \bar{y}^\pm).$$

Note, that it is easy to see that

$$\begin{aligned} \mathcal{H}^{\alpha\alpha'} \frac{\partial}{\partial y^{-\alpha}} \frac{\partial}{\partial \bar{y}^{-\alpha'}} f_- J(y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0} &= dH^{\mu\nu} \frac{\partial}{\partial y^{-\mu}} \frac{\partial}{\partial y^{-\nu}} J(y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0}, \\ \mathcal{H}^{\alpha\alpha'} \frac{\partial}{\partial y^{-\alpha}} \frac{\partial}{\partial \bar{y}^{-\alpha'}} f_+ J(y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0} &= d\bar{H}^{\mu'\nu'} \frac{\partial}{\partial \bar{y}^{-\mu'}} \frac{\partial}{\partial \bar{y}^{-\nu'}} J(y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0}, \end{aligned} \quad (3.23)$$

where $f_\pm \in sl_2$ (3.18).

Now we are in a position to consider a deformation of the system (2.6), (2.7) by solutions of the rank-two equations (3.11) and hence by arbitrary bilinear conformal currents.

4 Current deformation

To describe the current deformation of free $4d$ massless field equations we look for a nontrivial deformation of the combination of the rank-one unfolded system (2.6), (2.7) with the rank-two unfolded system (3.11). The form of the deformation is fixed by its formal consistency. The problem is solved in two steps.

Firstly, we consider the zero-form sector to find a nontrivial gluing of the rank-two current module to the rank-one Weyl module. The final result is presented in Subsection 4.1 while details of derivation are given in Appendix A. Secondly, the final result for the gluing in the one-form sector is presented in Subsection 4.2, while details are given in Appendices B and C.

4.1 Current deformation in the zero-form sector

Consider the following deformation of the equations (2.7)

$$\begin{aligned} D^{tw} C(y, \bar{y} | x) + e^{\alpha\alpha'} y_\alpha \left(F^+(N_\pm, \bar{N}_\pm) \frac{\partial}{\partial \bar{y}^{+\alpha'}} + F^-(N_\pm, \bar{N}_\pm) \frac{\partial}{\partial \bar{y}^{-\alpha'}} \right) \bar{J}(y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0} + \\ + e^{\alpha\alpha'} \bar{y}_{\alpha'} \left(\bar{F}^+(N_\pm, \bar{N}_\pm) \frac{\partial}{\partial y^{+\alpha}} + \bar{F}^-(N_\pm, \bar{N}_\pm) \frac{\partial}{\partial y^{-\alpha}} \right) J(y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0} = 0, \end{aligned} \quad (4.1)$$

where $J(y^\pm, \bar{y}^\pm)$, $\bar{J}(y^\pm, \bar{y}^\pm)$ satisfy rank-two unfolded field equations (3.11) and F^\pm are gluing operators to be found from the consistency condition for the system (4.1), which is analyzed in detail in Appendix A. The final result is

$$F^\pm(N_\pm, \bar{N}_\pm) = \sum_{n_\pm \geq -1} \pm \frac{\partial}{\partial N_\pm} \mathcal{F}_{n_-, n_+}(N_\pm, \bar{N}_\pm), \quad (4.2)$$

$$\bar{F}^\pm(N_\pm, \bar{N}_\pm) = \sum_{\bar{n}_\pm \geq -1} \pm \frac{\partial}{\partial \bar{N}_\pm} \bar{\mathcal{F}}_{\bar{n}_-, \bar{n}_+}(N_\pm, \bar{N}_\pm), \quad (4.3)$$

where

$$N_\pm = y^\alpha \frac{\partial}{\partial y^{\pm\alpha}}, \quad \bar{N}_\pm = \bar{y}^{\alpha'} \frac{\partial}{\partial \bar{y}^{\pm\alpha'}} \quad (4.4)$$

and for any coefficients A_{n_-, n_+} ,

$$\mathcal{F}_{n_-, n_+}(N_\pm, \bar{N}_\pm) = A_{n_-, n_+} (N_+)^{n_++1} (N_-)^{n_-+1} \sum_{m \geq 0} \frac{(-1)^m (\bar{N}_+ N_- + \bar{N}_- N_+)^m}{m! (n_+ + n_- + m + 3)!}, \quad (4.5)$$

$$\bar{\mathcal{F}}_{\bar{n}_-, \bar{n}_+}(N_\pm, \bar{N}_\pm) = \bar{A}_{\bar{n}_-, \bar{n}_+} (\bar{N}_+)^{\bar{n}_++1} (\bar{N}_-)^{\bar{n}_-+1} \sum_{m \geq 0} \frac{(-1)^m (\bar{N}_+ N_- + \bar{N}_- N_+)^m}{m! (\bar{n}_+ + \bar{n}_- + m + 3)!}. \quad (4.6)$$

4.2 Current deformation in the one-form sector

Since zero-forms contribute to the right-hand-sides of the equations (2.6), their formal consistency in presence of the deformation (4.1) in the zero-form sector requires an appropriate deformation in the one-form sector. The final result is

$$\begin{aligned} D^{ad} \omega(y, \bar{y}|x) &= \bar{H}^{\alpha' \beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \bar{C}(0, \bar{y} | x) + H^{\alpha \beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0 | x) + \\ &+ \bar{H}^{\alpha' \beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \bar{G}(N_\pm, \bar{N}_\pm) \bar{\mathcal{J}}(y^\pm, \bar{y}^\pm | x) \big|_{y^\pm = \bar{y}^\pm = 0} + \\ &+ H^{\alpha \beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} G(N_\pm, \bar{N}_\pm) \mathcal{J}(y^\pm, \bar{y}^\pm | x) \big|_{y^\pm = \bar{y}^\pm = 0}, \end{aligned} \quad (4.7)$$

$$D^{tw} C(y, \bar{y} | x) + e^{\alpha \alpha'} y_\alpha F^A(N_\pm, \bar{N}_\pm) \frac{\partial}{\partial \bar{y}^{A \alpha'}} \bar{\mathcal{J}}(y^\pm, \bar{y}^\pm | x) \big|_{y^\pm = \bar{y}^\pm = 0} = 0, \quad (4.8)$$

$$D^{tw} \bar{C}(y, \bar{y} | x) + e^{\alpha \alpha'} \bar{y}_\alpha \bar{F}^A(N_\pm, \bar{N}_\pm) \frac{\partial}{\partial y^{A \alpha}} \mathcal{J}(y^\pm, \bar{y}^\pm | x) \big|_{y^\pm = \bar{y}^\pm = 0} = 0,$$

where the fields $\mathcal{J}(y^\pm, \bar{y}^\pm | x)$, $\bar{\mathcal{J}}(y^\pm, \bar{y}^\pm | x)$, $\bar{\mathcal{J}}(y^\pm, \bar{y}^\pm | x)$ and $J(y^\pm, \bar{y}^\pm | x)$ and the operators F^+ , F^- , G and \bar{G} are defined below.

Given integer $2s$, F^\pm are given by (4.2) with $A = 1$, $n_+ = -1$ and $n_- = 2s - 1$

$$F^+ = (N_-)^{2s} \bar{N}_- \sum_{\bar{n}_+, \bar{n}_- \geq 0} (\tilde{N}_+)^{\bar{n}_+} (\tilde{N}_-)^{\bar{n}_-} \frac{(-1)^{\bar{n}_- + \bar{n}_+}}{\bar{n}_-! \bar{n}_+! (\bar{n}_- + \bar{n}_+ + 2s + 2)!}, \quad (4.9)$$

$$F^- = (N_-)^{2s-1} \sum_{\bar{n}_+, \bar{n}_- \geq 0} (\tilde{N}_+)^{\bar{n}_+} (\tilde{N}_-)^{\bar{n}_-} \frac{(-1)^{\bar{n}_- + \bar{n}_+} (\bar{n}_+ + 2s)}{\bar{n}_-! \bar{n}_+! (\bar{n}_- + \bar{n}_+ + 2s + 1)!}. \quad (4.10)$$

Since the constructions of one-form deformed equations is different for integer and half-integer cases they will be described separately.

For the future convenience we will use the following decompositions

$$A(y^\pm, \bar{y}^\pm | x) = \sum_{m, \bar{m}} A^{m, \bar{m}}(y^\pm, \bar{y}^\pm | x), \quad A(y, \bar{y} | x) = \sum_{m, \bar{m}} A^{m, \bar{m}}(y, \bar{y} | x), \quad (4.11)$$

where

$$\begin{aligned}
M_{+-} A^{m,\bar{m}}(y^\pm, \bar{y}^\pm | x) &= m A^{m,\bar{m}}(y^\pm, \bar{y}^\pm | x), & \overline{M}_{+-} A^{m,\bar{m}}(y^\pm, \bar{y}^\pm | x) &= \bar{m} A^{m,\bar{m}}(y^\pm, \bar{y}^\pm | x) \\
M A^{m,\bar{m}}(y, \bar{y} | x) &= m A^{m,\bar{m}}(y, \bar{y} | x), & \overline{M} A^{m,\bar{m}}(y, \bar{y} | x) &= \bar{m} A^{m,\bar{m}}(y, \bar{y} | x), \\
M_{+-} &= y^{+\beta} \frac{\partial}{\partial y^{+\beta}} + y^{-\beta} \frac{\partial}{\partial y^{-\beta}}, & \overline{M}_{+-} &= \bar{y}^{+\beta'} \frac{\partial}{\partial \bar{y}^{+\beta'}} + \bar{y}^{-\beta'} \frac{\partial}{\partial \bar{y}^{-\beta'}}, \\
M &= y^\beta \frac{\partial}{\partial y^\beta}, & \overline{M} &= \bar{y}^{\beta'} \frac{\partial}{\partial \bar{y}^{\beta'}}.
\end{aligned}$$

4.2.1 Integer spin

In Appendix B it is shown that the following functions and operators ensure the consistency of Eqs. (4.7), (4.8) for given integer spin $s \geq 2$ case:

$$G = \sum_{k=2}^s \frac{(-1)^k (N_-)^k (\overline{N}_-)^{2s-k}}{(2s-k)!k!}, \quad \overline{G} = \sum_{k=2}^s \frac{(-1)^k (N_-)^{2s-k} (\overline{N}_-)^k}{(2s-k)!k!}, \quad (4.12)$$

$$\begin{aligned}
\mathcal{J} &= (-1)^s \sum_{k=0}^{s-2} \left(f_-\right)^k \tilde{\mathcal{J}}_0, & \overline{\mathcal{J}} &= (-1)^{(s)} \sum_{k=0}^{s-2} \left(-f_+\right)^k \tilde{\mathcal{J}}_0, \\
\overline{\mathcal{J}} &= \lambda (-1)^{(s)} \left(-f_+\right)^{s-1} \tilde{\mathcal{J}}_0, & J &= \lambda (-1)^s \left(f_-\right)^{s-1} \tilde{\mathcal{J}}_0,
\end{aligned} \quad (4.13)$$

where a *primary current field* $\tilde{\mathcal{J}}_0 = \overline{\tilde{\mathcal{J}}}_0$ is any solution of Eq. (3.11), while f_\pm are sl_2 generators (3.17).

Note that, in accordance with (4.11), the primary current fields $\tilde{\mathcal{J}}_0$, that give nonzero deformations to (4.8), are of the form

$$\tilde{\mathcal{J}}_0 = \sum_{n \geq 0} \tilde{\mathcal{J}}^{n,n}(y^\pm, \bar{y}^\pm | x). \quad (4.14)$$

4.2.2 Half-integer spin

In Appendix B, it is shown that the consistency of Eq. (4.7), (4.8) for the half-integer $s > 3/2$ case is provided by functions and operators described below. The case of $s = 3/2$ is special and is considered in Appendix C. However, the final result is described by the same formulae for all $s \geq 3/2$.

We set

$$\begin{aligned}
G &= \sum_{k=2}^{[s]} \frac{(-1)^k (N_-)^k (\overline{N}_-)^{2s-k}}{(2s-k)!k!} + \sum_{k=2}^{[s]+1} \frac{(-1)^k (N_-)^k (\overline{N}_-)^{2s-k}}{(2s-k)!k!}, \\
\overline{G} &= \sum_{k=2}^{[s]} \frac{(-1)^k (N_-)^{2s-k} (\overline{N}_-)^k}{(2s-k)!k!} + \sum_{k=2}^{[s]+1} \frac{(-1)^k (N_-)^{2s-k} (\overline{N}_-)^k}{(2s-k)!k!},
\end{aligned} \quad (4.15)$$

and and

$$\begin{aligned}
\mathcal{J} &= \sum_{k=0}^{[s]-2} (f_-)^k \tilde{\mathcal{J}}_{-\frac{1}{2}} \sum_{k=0}^{[s]-1} (f_-)^k \tilde{\mathcal{J}}_{+\frac{1}{2}}, \\
\overline{\mathcal{J}} &= - \sum_{k=0}^{[s]-1} (-f_+)^k \tilde{\mathcal{J}}_{-\frac{1}{2}} + - \sum_{k=0}^{[s]-2} (-f_+)^k \tilde{\mathcal{J}}_{+\frac{1}{2}}, \\
J &= \lambda \overline{A}^{-1}(-1)^{[s]} (f_-)^{[s]-1} \tilde{\mathcal{J}}_{-\frac{1}{2}} + \lambda \overline{A}^{-1}(-1)^{[s]+1} (f_-)^{[s]} \tilde{\mathcal{J}}_{+\frac{1}{2}}, \\
\overline{J} &= \lambda A^{-1}(-1)^{[s]} (-f_+)^{[s]} \tilde{\mathcal{J}}_{-\frac{1}{2}} + \lambda A^{-1}(-1)^{[s]+1} (-f_+)^{[s]-1} \tilde{\mathcal{J}}_{+\frac{1}{2}}.
\end{aligned} \tag{4.16}$$

where $s \geq 3/2$ and *primary current fields* $\tilde{\mathcal{J}}_{+\frac{1}{2}} = -\overline{\tilde{\mathcal{J}}_{-\frac{1}{2}}}$ are solutions of Eq. (3.11). Note that for spin $s = 3/2$ the (4.16)- (4.15), that contain $\sum_{k=m}^{m-1}(\dots)$ should be set equal zero.

Note that, in accordance with (4.11), for given half-integer s the primary current fields $\tilde{\mathcal{J}}_{\pm\frac{1}{2}}$, that give nonzero deformation to (4.8), are of the form

$$\tilde{\mathcal{J}}_{-\frac{1}{2}} = \sum_{n \geq 0} \tilde{\mathcal{J}}^{n,n+1}(y^\pm, \bar{y}^\pm | x), \quad \tilde{\mathcal{J}}_{+\frac{1}{2}} = \sum_{n \geq 0} \tilde{\mathcal{J}}^{n+1,n}(y^\pm, \bar{y}^\pm | x). \tag{4.17}$$

5 Current contributions to dynamical equations

Let us show how the constructed unfolded equations affect the form of dynamical equations for massless fields contained in the unfolded system. To obtain usual current interactions, the rank-two fields should be realized as bilinears of massless fields

$$J(y^\pm, \bar{y}^\pm | x) = \mathcal{C}_+ \left(y^+ + y^-, \bar{y}^+ + \bar{y}^- | x \right) \mathcal{C}_- \left(y^+ - y^-, \bar{y}^+ - \bar{y}^- | x \right), \tag{5.1}$$

where rank-one fields $\mathcal{C}_\pm(\frac{1}{\sqrt{2}}y, \frac{1}{\sqrt{2}}\bar{y} | x)$ solve the rank-one equations (3.15).

5.1 Spin 0

For $s = 0$, Eq. (4.1) gives

$$D^L_{\alpha\alpha'} C(0, 0 | x) + \lambda C_{\alpha\alpha'}(0, 0 | x) = 0, \tag{5.2}$$

$$\begin{aligned}
&D^L_{\alpha\alpha'} C_{\beta\beta'}(0, 0 | x) + \lambda C_{\alpha\beta\alpha'\beta'}(0, 0 | x) + \lambda \varepsilon_{\alpha'\beta'} \varepsilon_{\alpha\beta} C(0, 0 | x) + \\
&- \frac{\varepsilon_{\alpha'\beta'}}{2} \left(\frac{\partial}{\partial y^{+\beta}} \frac{\partial}{\partial y^{-\alpha}} - \frac{\partial}{\partial y^{-\beta}} \frac{\partial}{\partial y^{+\alpha}} \right) J(y^\pm, 0) \Big|_{y^\pm = \bar{y}^\pm = 0} + \\
&- \frac{\varepsilon_{\alpha\beta}}{2} \left(\frac{\partial}{\partial \bar{y}^{+\beta'}} \frac{\partial}{\partial \bar{y}^{-\alpha'}} - \frac{\partial}{\partial \bar{y}^{-\beta'}} \frac{\partial}{\partial \bar{y}^{+\alpha'}} \right) \overline{J}(0, \bar{y}^\pm) \Big|_{y^\pm = \bar{y}^\pm = 0} = 0,
\end{aligned}$$

where $J(y^\pm, \bar{y}^\pm|x) = \bar{J}(y^\pm, \bar{y}^\pm|x)$ satisfy the current equations (3.11). Hence

$$D_{\alpha\alpha'}^L D^{L\alpha\alpha'} C(0, 0|x) = 4\lambda^2 C(0, 0|x) - 4 \frac{\partial}{\partial y_{+\alpha}^+} \frac{\partial}{\partial y_{-\alpha}^-} J(y^\pm, 0) - 4 \frac{\partial}{\partial \bar{y}_{+\alpha'}^+} \frac{\partial}{\partial \bar{y}_{-\alpha'}^-} \bar{J}(0, \bar{y}^\pm), \quad (5.3)$$

Using the bilinear formula (5.1) we obtain

$$D_{\alpha\alpha'}^L D^{L\alpha\alpha'} C(0, 0|x) = 4\lambda^2 C(0, 0|x) + 4\bar{\mathcal{C}}_{+\alpha'}(x)\bar{\mathcal{C}}_{-\alpha'}(x) + 4\mathcal{C}_{+\alpha}(x)\mathcal{C}_{-\alpha}(x). \quad (5.4)$$

Remarkably, in the spin zero sector, the general unfolded construction reproduces the usual Yukawa interaction. Note that, a \mathcal{C}^2 deformation, that one might naively expect in the spin zero sector, does not appear. This is consistent with the fact that the construction of this paper is conformal, while the possible \mathcal{C}^2 deformation is not conformal in four dimensions.

5.2 Spin 1/2

For given solutions $J(y^\pm, \bar{y}^\pm|x) = \bar{J}(y^\pm, \bar{y}^\pm|x)$ of the current equations (3.11) Eq. (4.1) give

$$\begin{aligned} D_{\alpha\alpha'}^L C_\mu(0, 0|x) + \lambda C_{\mu\alpha\alpha'}(0, 0|x) + \frac{1}{2}\varepsilon_{\mu\alpha} \frac{\partial}{\partial \bar{y}_{+\alpha'}^+} \bar{J}(0, \bar{y}^\pm) &= 0, \\ D_{\alpha\alpha'}^L C_{\mu'}(0, 0|x) + \lambda C_{\alpha\mu'\alpha'}(0, 0|x) + \frac{1}{2}\varepsilon_{\mu'\alpha'} \frac{\partial}{\partial y_{+\alpha}^+} J(y^\pm, 0) &= 0. \end{aligned} \quad (5.5)$$

From (5.5) it follows

$$D_{\alpha\alpha'}^L C^\alpha(0, 0|x) - \frac{\partial}{\partial \bar{y}_{-\alpha'}^-} \bar{J}(0, \bar{y}^\pm) = 0, \quad D_{\alpha\alpha'}^L C^{\alpha'}(0, 0|x) - \frac{\partial}{\partial y_{-\alpha}^-} J(y^\pm, 0) = 0. \quad (5.6)$$

Substitution of bilinear $J \bar{J}$ (5.1) built from fermions and bosons, gives

$$\begin{aligned} D_{\alpha\alpha'}^L C^\alpha(x) - 2^{-\frac{1}{2}}\bar{\mathcal{C}}_{+\alpha'}(x)\mathcal{C}_-(x) + 2^{-\frac{1}{2}}\bar{\mathcal{C}}_{+\alpha'}(x)\mathcal{C}_-(x) &= 0, \\ D_{\alpha\alpha'}^L \bar{\mathcal{C}}^{\alpha'}(x) - 2^{-\frac{1}{2}}\mathcal{C}_{+\alpha}(x)\mathcal{C}_-(x) + 2^{-\frac{1}{2}}\mathcal{C}_{-\alpha}(x)\mathcal{C}_+(x) &= 0, \end{aligned} \quad (5.7)$$

which is again the Yukawa interaction but now in the spin 1/2 sector.

5.3 Maxwell equations

For any solutions $J(y^\pm, \bar{y}^\pm|x) = \bar{J}(y^\pm, \bar{y}^\pm|x)$ of the current equations (3.11), Eq. (2.6) still reads as

$$D^{ad}\omega(x) = \bar{H}^{\alpha'\beta'} \bar{\mathcal{C}}_{\alpha'\beta'}(x) + H^{\alpha\beta} C_{\alpha\beta}(x). \quad (5.8)$$

This identifies $C_{\alpha\beta}(x)$ and $\bar{\mathcal{C}}_{\alpha'\beta'}(x)$ with selfdual and antiselfdual parts of the Maxwell field strength. The consistency conditions of (5.8) imply the Bianchi identities

$$D^{ad}(H^{\alpha\beta} C_{\alpha\beta}(x) + \bar{H}^{\alpha'\beta'} \bar{\mathcal{C}}_{\alpha'\beta'}(x)) = 0. \quad (5.9)$$

Deformed Eq. (4.1) reads as

$$D^L_{\alpha\alpha'} C_{\mu\nu}(0, 0|x) + \lambda C_{\mu\nu\alpha\alpha'}(0, 0|x) + \quad (5.10)$$

$$+ \frac{1}{3} \varepsilon_{\mu\alpha} \frac{\partial}{\partial \bar{y}^{-\alpha'}} \frac{\partial}{\partial y^{-\nu}} J(y^\pm, \bar{y}^\pm) \Big|_{y^\pm = \bar{y}^\pm = 0} + \frac{1}{3} \varepsilon_{\nu\alpha} \frac{\partial}{\partial \bar{y}^{-\alpha'}} \frac{\partial}{\partial y^{-\mu}} J(y^\pm, \bar{y}^\pm) \Big|_{y^\pm = \bar{y}^\pm = 0},$$

where the gluing operators F^\pm in (4.1) are of the form (4.9), (4.10) with $s = 1$. From (5.10) it follows that, in accordance with the decompositions (4.11),

$$D^L_{\alpha'} C_{\mu\nu}(0, 0|x) + \frac{\partial}{\partial \bar{y}^{-\alpha'}} \frac{\partial}{\partial y^{-\nu}} J^{1,1}(y^\pm, \bar{y}^\pm) = 0. \quad (5.11)$$

By virtue of (5.11) along with the identities

$$H^{\alpha\beta} \wedge e^{\mu\mu'} = \epsilon^{\alpha\mu} \mathcal{H}^{\beta\mu'} + \epsilon^{\beta\mu} \mathcal{H}^{\alpha\mu'}, \quad \bar{H}^{\alpha'\beta'} \wedge e^{\mu\mu'} = -\epsilon^{\alpha'\mu'} \mathcal{H}^{\mu\beta'} - \epsilon^{\beta'\mu'} \mathcal{H}^{\mu\alpha'}, \quad (5.12)$$

we have

$$H^{\alpha\beta} e^{\nu\nu'} D^{ad}_{\nu\nu'} C_{\alpha\beta}(x) = 2\mathcal{H}^{\beta\nu'} D^{ad\alpha}_{\nu'} C_{\alpha\beta} = -2\mathcal{H}^{\beta\nu'} \frac{\partial}{\partial y^{-\beta}} \frac{\partial}{\partial y^{-\nu'}} J^{1,1}(y^\pm, \bar{y}^\pm).$$

Analogously,

$$\bar{H}^{\alpha'\beta'} D^{ad} \bar{C}_{\alpha'\beta'}(x) = +2\mathcal{H}^{\beta\nu'} \frac{\partial}{\partial y^{-\beta}} \frac{\partial}{\partial y^{-\nu'}} J^{1,1}(y^\pm, \bar{y}^\pm).$$

Hence it follows that, as anticipated, the Bianchi identities (5.9) are respected and

$$D^{ad} \left(H^{\alpha\beta} C_{\alpha\beta}(x) - \bar{H}^{\alpha'\beta'} \bar{C}_{\alpha'\beta'}(x) \right) = -4\mathcal{H}^{\beta\nu'} \frac{\partial}{\partial y^{-\beta}} \frac{\partial}{\partial y^{-\nu'}} J^{1,1}(y^\pm, \bar{y}^\pm). \quad (5.13)$$

Eq.(5.13) is just the Maxwell equations with nonzero current.

For J (5.1) built from scalars we have

$$2\mathcal{H}^{\beta\nu'} \frac{\partial}{\partial y^{-\beta}} \frac{\partial}{\partial y^{-\nu'}} J^{1,1}(y^\pm, \bar{y}^\pm|x) = \mathcal{H}^{\beta\nu'} \left(-\mathcal{C}_-(x) \frac{\partial}{\partial x^{\beta\nu'}} \mathcal{C}_+(x) + \mathcal{C}_+(x) \frac{\partial}{\partial x^{\beta\nu'}} \mathcal{C}_-(x) \right),$$

while for J (5.1) built from fermions

$$\mathcal{H}^{\beta\nu'} \frac{\partial}{\partial y^{-\beta}} \frac{\partial}{\partial y^{-\nu'}} J^{1,1}(y^\pm, \bar{y}^\pm|x) = -\mathcal{H}^{\beta\nu'} \mathcal{C}_{+\beta}(x) \bar{\mathcal{C}}_{-\nu'}(x),$$

which are the standard expressions for the spin one currents.

5.4 Spin 3/2

Using (4.11), from Eq. (4.7) along with (4.15) and (4.16) we have

$$\begin{aligned} D^L \omega^{0,1}(0, \bar{y}) - \lambda e^{\beta\beta'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\beta} \omega^{1,0}(y, 0|x) = \\ = \bar{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \bar{C}(0, \bar{y}|x) + 2H^{\alpha\beta} \frac{\partial^2}{\partial y^{-\alpha} \partial y^{-\beta}} \mathcal{J}^{2,1}(y^\pm, \bar{y}^\pm|x), \end{aligned} \quad (5.14)$$

$$\begin{aligned} D^L \omega^{1,0}(y, 0|x) - \lambda e^{\beta\beta'} y_\beta \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{0,1}(0, \bar{y}|x) = \\ = H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0|x) + 2\bar{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{-\alpha'} \partial \bar{y}^{-\beta'}} \bar{\mathcal{J}}^{1,2}(y^\pm, \bar{y}^\pm|x). \end{aligned} \quad (5.15)$$

Setting $\omega^{0,1} = e^{\alpha\beta'} \omega^{0,1}_{\alpha\beta'}$ and $\omega^{1,0} = e^{\alpha\beta'} \omega^{1,0}_{\alpha\beta'}$, from (5.14), (5.15) we obtain spin 3/2 massless equations in AdS_4 in the form

$$\begin{aligned} D^L_{\beta\beta'} \omega^{0,1}_{\alpha\beta'}(0, \bar{y}) - \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\beta} \omega^{1,0}_{\alpha\beta'}(y, 0|x) = 2 \frac{\partial^2}{\partial y^{-\alpha} \partial y^{-\beta}} \mathcal{J}^{2,1}(y^\pm, \bar{y}^\pm|x), \\ D^L_{\beta\beta'} \omega^{1,0}_{\alpha\beta'}(y, 0|x) - \lambda y_\beta \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{0,1}_{\alpha\beta'}(0, \bar{y}|x) = 2 \frac{\partial^2}{\partial \bar{y}^{-\alpha'} \partial \bar{y}^{-\beta'}} \bar{\mathcal{J}}^{1,2}(y^\pm, \bar{y}^\pm|x). \end{aligned} \quad (5.16)$$

Substitution of bilinear J (5.1) gives

$$\begin{aligned} \frac{\partial}{\partial \bar{y}^{-\nu'}} D^L_{\alpha\beta'} \omega^{0,1}_{\alpha\beta'}(0, \bar{y}) + \lambda \frac{\partial}{\partial y^\alpha} \omega^{1,0}_{\alpha\nu'}(y, 0|x) = \\ = \sqrt{2} \left(-\mathcal{C}^{2,0}_{+\alpha\alpha}(0, 0|x) \bar{\mathcal{C}}^{0,1}_{-\nu'}(0, 0|x) - \mathcal{C}^{0,0}_{+\alpha\nu'}(0, 0|x) \mathcal{C}^{1,0}_{-\alpha}(0, 0|x) \right) + (+ \leftrightarrow -), \\ \frac{\partial}{\partial y^{-\nu}} D^L_{\beta\beta'} \omega^{1,0}_{\alpha\beta'}(y, 0|x) + \lambda \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{0,1}_{\nu\alpha'}(0, \bar{y}|x) = \\ = \sqrt{2} \left(-\bar{\mathcal{C}}^{0,2}_{+\alpha'\alpha'}(0, 0|x) \mathcal{C}^{1,0}_{-\nu}(0, 0|x) - \bar{\mathcal{C}}^{0,0}_{+\nu\alpha'}(0, 0|x) \bar{\mathcal{C}}^{0,1}_{-\alpha'}(0, 0|x) \right) (+ \leftrightarrow -). \end{aligned} \quad (5.17)$$

This is the Rarita-Schwinger equation with supercurrent built of a scalar and spinor.

5.5 Spin two

In the case of $s = 2$ we obtain from Eq. (4.7)

$$\begin{aligned} D^{ad} \omega(y, \bar{y}|x) = \bar{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \bar{C}(0, \bar{y} | x) + H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0 | x) + \\ + \frac{1}{12} \bar{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} (N_-)^2 (\bar{N}_-)^2 \mathcal{J} \Big|_{y^\pm = \bar{y}^\pm = 0} + \frac{1}{12} H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} (N_-)^2 (\bar{N}_-)^2 \mathcal{J} \Big|_{y^\pm = \bar{y}^\pm = 0}. \end{aligned} \quad (5.18)$$

In accordance with the decompositions (4.11), this gives

$$D^L \omega^{1,1}(y, \bar{y}|x) = \lambda e^{\alpha\beta'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{2,0}(y, 0|x) + \lambda e^{\alpha\beta'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{0,2}(0, \bar{y}|x), \quad (5.19)$$

$$\begin{aligned} D^L \omega^{0,2}(0, \bar{y}) &= \lambda e^{\alpha\beta'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{1,1}(y, \bar{y}|x) + \bar{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \bar{C}(0, \bar{y}|x) + \\ &+ \frac{1}{3} H^{\alpha\beta} \bar{y}^{\alpha'} \bar{y}^{\beta'} \frac{\partial^2}{\partial \bar{y}^{-\alpha'} \partial \bar{y}^{-\beta'}} \frac{\partial^2}{\partial y^{-\alpha} \partial y^{-\beta}} \mathcal{J}^{2,2}(y^-, \bar{y}^-|x), \end{aligned} \quad (5.20)$$

$$\begin{aligned} D^L \omega^{2,0}(y, 0|x) &= \lambda e^{\alpha\beta'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{1,1}(y, \bar{y}|x) + H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0|x) + \\ &+ \frac{1}{3} \bar{H}^{\alpha'\beta'} y^\alpha y^\beta \frac{\partial^2}{\partial y^{-\alpha} \partial y^{-\beta}} \frac{\partial^2}{\partial \bar{y}^{-\alpha'} \partial \bar{y}^{-\beta'}} \mathcal{J}^{2,2}(y^-, \bar{y}^-|x). \end{aligned} \quad (5.21)$$

Eq. (5.19) gives

$$D^L_{\beta\beta'} \omega^{1,1}_{\beta^{\beta'}}(y, \bar{y}|x) = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\beta} \omega^{2,0}_{\beta^{\beta'}}(y, 0|x) + \lambda y_\beta \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{0,2}_{\beta^{\beta'}}(0, \bar{y}|x), \quad (5.22)$$

$$D^L_{\beta\beta'} \omega^{1,1\beta}_{\beta'}(y, \bar{y}|x) = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\beta} \omega^{2,0\beta}_{\beta'}(y, 0|x) + \lambda y_\beta \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{0,2\beta}_{\beta'}(0, \bar{y}|x). \quad (5.23)$$

From Eq. (5.20) we obtain

$$D^L_{\beta\beta'} \omega^{0,2}_{\beta^{\beta'}} = \frac{1}{3} \bar{y}^{\alpha'} \bar{y}^{\beta'} \frac{\partial^2}{\partial \bar{y}^{-\alpha'} \partial \bar{y}^{-\beta'}} \frac{\partial^2}{\partial y^{-\beta} \partial y^{-\beta}} \mathcal{J}^{2,2} + \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\beta} \omega^{1,1}_{\beta^{\beta'}}, \quad (5.24)$$

$$D^L_{\beta\beta'} \omega^{2,0\beta}_{\beta'} = \frac{1}{3} y^\alpha y^\beta \frac{\partial^2}{\partial y^{-\alpha} \partial y^{-\beta}} \frac{\partial^2}{\partial \bar{y}^{-\beta'} \partial \bar{y}^{-\beta'}} \mathcal{J}^{2,2} + \lambda y_\beta \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{1,1\beta}_{\beta'}. \quad (5.25)$$

The equations (5.22) and (5.23) express the Lorentz connection $\omega^{2,0}$ and $\omega^{0,2}$ via derivatives of the vierbein $\omega^{1,1}$ while the equations (5.24) and (5.25) contain the Bianchi identities for Eq. (5.19)

$$\frac{\partial^2}{\partial \bar{y}^{\nu'} \partial \bar{y}^{\nu'}} D^L_{\beta\beta'} \omega^{0,2}_{\beta^{\beta'}} = \frac{\partial^2}{\partial y^\beta \partial y^\beta} D^L_{\nu\nu'} \omega^{2,0\nu}_{\nu'}, \quad (5.26)$$

and linearized Einstein equations

$$\frac{\partial^2}{\partial \bar{y}^{\nu'} \partial \bar{y}^{\nu'}} D^L_{\beta\beta'} \omega^{0,2}_{\beta^{\beta'}} - 2\lambda \frac{\partial}{\partial \bar{y}^{\nu'}} \frac{\partial}{\partial y^\beta} \omega^{1,1}_{\beta\nu'} = \frac{2}{3} \frac{\partial^2}{\partial \bar{y}^{-\nu'} \partial \bar{y}^{-\nu'}} \frac{\partial^2}{\partial y^{-\beta} \partial y^{-\beta}} \mathcal{J}^{2,2}, \quad (5.27)$$

that contain the contribution of the stress tensor.

Substitution of the bilinear J (5.1) gives

$$\begin{aligned} &\frac{\partial^2}{\partial \bar{y}^{\nu'} \partial \bar{y}^{\nu'}} D^L_{\beta\beta'} \omega^{0,2}_{\beta^{\beta'}} - 2\lambda \frac{\partial}{\partial \bar{y}^{\nu'}} \frac{\partial}{\partial y^\beta} \omega^{1,1}_{\beta\nu'} = \\ &= \frac{2}{3} \left(\mathcal{C}^{2,0}_{+\alpha\alpha}(0, 0|x) \bar{\mathcal{C}}^{0,2}_{-\alpha'\alpha'}(0, 0|x) + \mathcal{C}^{1,0}_{+\alpha\alpha\alpha'}(0, 0|x) \bar{\mathcal{C}}^{0,1}_{-\alpha'}(0, 0|x) + \right. \\ &\quad \left. + \mathcal{C}^{0,0}_{+\alpha\alpha'}(0, 0|x) \bar{\mathcal{C}}^{0,0}_{-\alpha\alpha'}(0, 0|x) + (+ \leftrightarrow -) \right), \end{aligned}$$

containing the stress tensor built from the massless fields of spins 0, 1/2 and 1.

5.6 Arbitrary spin

5.6.1 Integer spins

For any integer s and real current, that satisfies $\tilde{\mathcal{J}}_0 = \overline{\tilde{\mathcal{J}}}_0$, decomposing ω as in (4.11) we obtain from (4.7) using (4.12) and (4.13)

$$D^L \omega^{s-1, s-1}(y, \bar{y}|x) = \lambda e^{\alpha\beta'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s, s-2}(y, \bar{y}|x) + \lambda e^{\alpha\beta'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-2, s}(y, \bar{y}|x), \quad (5.28)$$

$$\begin{aligned} D^L \omega^{s, s-2}(y, \bar{y}|x) &= \lambda e^{\alpha\beta'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-1, s-1}(y, \bar{y}|x) + \\ &+ \lambda e^{\alpha\beta'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s+1, s-3}(y, \bar{y}|x) + \overline{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \frac{(-1)^s}{(s)!s(s-1)} (N_-)^s (\overline{N}_-)^s \tilde{\mathcal{J}}_0^{s, s}, \end{aligned} \quad (5.29)$$

$$\begin{aligned} D^L \omega^{s-2, s}(y, \bar{y}|x) &= \lambda e^{\alpha\beta'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s-1, s-1}(y, \bar{y}|x) + \\ &+ \lambda e^{\alpha\beta'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-3, s+1}(y, \bar{y}|x) + H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} \frac{(-1)^s}{(s)!s(s-1)} (N_-)^s (\overline{N}_-)^s \tilde{\mathcal{J}}_0^{s, s}. \end{aligned} \quad (5.30)$$

From here it follows that

$$e^{\mu\mu'} e^{\nu\nu'} D_{\mu\mu'}^L \omega^{s-1, s-1}_{\nu\nu'} = \lambda e^{\alpha\beta'} e^{\nu\nu'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s, s-2}_{\nu\nu'} + \lambda e^{\alpha\beta'} e^{\nu\nu'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-2, s}_{\nu\nu'}, \quad (5.31)$$

$$\begin{aligned} e^{\mu\mu'} e^{\nu\nu'} D_{\mu\mu'}^L \omega^{s, s-2}_{\nu\nu'} &= \lambda e^{\alpha\beta'} e^{\nu\nu'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-1, s-1}_{\nu\nu'} + \\ &+ \lambda e^{\alpha\beta'} e^{\nu\nu'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s+1, s-3}_{\nu\nu'} + \overline{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \frac{(-1)^s}{(s)!s(s-1)} (N_-)^s (\overline{N}_-)^s \tilde{\mathcal{J}}_0^{s, s}, \end{aligned} \quad (5.32)$$

$$\begin{aligned} e^{\mu\mu'} e^{\nu\nu'} D_{\mu\mu'}^L \omega^{s-2, s}_{\nu\nu'} &= \lambda e^{\alpha\beta'} e^{\nu\nu'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s-1, s-1}_{\nu\nu'} + \\ &+ \lambda e^{\alpha\beta'} e^{\nu\nu'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-3, s+1}_{\nu\nu'} + H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} \frac{(-1)^s}{(s)!s(s-1)} (N_-)^s (\overline{N}_-)^s \tilde{\mathcal{J}}_0^{s, s}. \end{aligned} \quad (5.33)$$

Hence

$$D_{\alpha\mu}^L \omega^{s-1, s-1}_{\alpha}{}^{\mu'} = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s, s-2}_{\alpha}{}^{\beta'} + \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-2, s}_{\alpha}{}^{\beta'}, \quad (5.34)$$

$$D_{\mu\beta'}^L \omega^{s-1, s-1}_{\beta'}{}^{\mu} = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s, s-2}_{\beta'}{}^{\alpha} + \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-2, s}_{\beta'}{}^{\alpha},$$

$$D_{\alpha\mu}^L \omega^{s, s-2}_{\alpha}{}^{\mu'} = \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-1, s-1}_{\alpha}{}^{\beta'} + \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s+1, s-3}_{\alpha}{}^{\beta'}, \quad (5.35)$$

$$D_{\mu\beta'}^L \omega^{s-2, s}_{\beta'}{}^{\mu} = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s-1, s-1}_{\beta'}{}^{\alpha} + \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-3, s+1}_{\beta'}{}^{\alpha},$$

$$D_{\alpha\mu'}^L \omega^{s-2,s}{}_{\alpha}{}^{\mu'} = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s-1,s-1}{}_{\alpha}{}^{\beta'} + \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-3,s+1}{}_{\alpha}{}^{\beta'} + \quad (5.36)$$

$$+ \frac{\partial^2}{\partial y^\alpha \partial y^\alpha} \frac{(-1)^s}{(s)!s(s-1)} (N_-)^s (\bar{N}_-)^s \tilde{\mathcal{J}}_0^{s,s},$$

$$D_{\mu\beta'}^L \omega^{s,s-2\mu}{}_{\beta'} = \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-1,s-1\alpha}{}_{\beta'} + \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s+1,s-3\alpha}{}_{\beta'} + \quad (5.37)$$

$$\frac{\partial^2}{\partial \bar{y}^{\beta'} \partial \bar{y}^{\beta'}} \frac{(-1)^s}{(s)!s(s-1)} (N_-)^s (\bar{N}_-)^s \tilde{\mathcal{J}}_0^{s,s}.$$

Substitution of bilinear J, \bar{J} (5.1) gives

$$D_{\alpha\mu'}^L \omega^{s-2,s}{}_{\alpha}{}^{\mu'} = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s-1,s-1}{}_{\alpha}{}^{\beta'} + \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-3,s+1}{}_{\alpha}{}^{\beta'} + \quad (5.38)$$

$$+ \frac{\partial^2}{\partial y^\alpha \partial y^\alpha} \frac{(-1)^s}{(s)!s(s-1)} (N_-)^s (\bar{N}_-)^s \sum_{p=0}^s \left(\mathcal{C}_+^{p,0}(y, 0|x) \bar{\mathcal{C}}_-^{0,p}(0, \bar{y}|x) + \bar{\mathcal{C}}_+^{0,p}(0, \bar{y}|x) \mathcal{C}_-^{p,0}(y, 0|x) \right),$$

$$D_{\mu\beta'}^L \omega^{s,s-2\mu}{}_{\beta'} = \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{s-1,s-1\alpha}{}_{\beta'} + \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{s+1,s-3\alpha}{}_{\beta'} + \quad (5.39)$$

$$\frac{\partial^2}{\partial \bar{y}^{\beta'} \partial \bar{y}^{\beta'}} \frac{(-1)^s}{(s)!s(s-1)} (N_-)^s (\bar{N}_-)^s \sum_{p=0}^s \left(\mathcal{C}_+^{p,0}(y, 0|x) \bar{\mathcal{C}}_-^{0,p}(0, \bar{y}|x) + \bar{\mathcal{C}}_+^{0,p}(0, \bar{y}|x) \mathcal{C}_-^{p,0}(y, 0|x) \right).$$

To obtain the dynamical spin s equations with the current corrections it remains to project out the terms, that contain $\omega^{s-3,s+1}$ and $\omega^{s+1,s-3}$. This is achieved by the contraction of free indices in (5.38) with $y^\alpha y^\alpha$ and in (5.39) with $\bar{y}^{\beta'} \bar{y}^{\beta'}$.

These terms describe the contribution of HS currents of [10] to the right-hand-sides of Fronsdal's equations in AdS_4 . Let us note that the fact that the equations (5.35) end hence their similar projections give zero result is a manifestation of conformal invariance of the currents under consideration which do not contribute to the trace part of the Fronsdal equations.

5.6.2 Half-integer spins

Using (4.15) along with (4.16), (4.17) and decomposing ω as in (4.11), from (4.7) we obtain for a half-integer s

$$D^L \omega^{[s]-1, [s]}(y, \bar{y}|x) = \lambda e^{\alpha\beta'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{[s]-2, [s]+1}(y, \bar{y}|x) + \lambda e^{\alpha\beta'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{[s], [s]-1}(y, \bar{y}|x) + \quad (5.40)$$

$$+ H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} \frac{(-1)^{[s]+1} (N_-)^{[s]+1} (\bar{N}_-)^{[s]}}{([s]+1)! [s]} \mathcal{J}_{+\frac{1}{2}}^{[s]+1, [s]}(y^\pm, \bar{y}^\pm|x),$$

$$D^L \omega^{[s], [s]-1}(y, \bar{y}|x) = \lambda e^{\alpha\beta'} \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{[s]+1, [s]-2}(y, \bar{y}|x) + \lambda e^{\alpha\beta'} y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{[s]-1, [s]}(y, \bar{y}|x) + \quad (5.41)$$

$$\bar{H}^{\alpha'\beta'} \frac{\partial^2}{\partial \bar{y}^{\alpha'} \partial \bar{y}^{\beta'}} \frac{(-1)^{[s]+1} (N_-)^{[s]} (\bar{N}_-)^{[s]+1}}{([s]+1)! [s]} \bar{\mathcal{J}}_{-\frac{1}{2}}^{[s], [s]+1}(y^\pm, \bar{y}^\pm|x),$$

where $\mathcal{J}_{+\frac{1}{2}}$ satisfies $\overline{\mathcal{J}_{+\frac{1}{2}}} = \bar{\mathcal{J}}_{-\frac{1}{2}}$.

Hence

$$D_{\alpha\mu'}^L \omega^{[s]-1, [s]}_\alpha{}^{\mu'} = \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{[s]-2, [s]+1}_\alpha{}^{\beta'} + \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{[s], [s]-1}_\alpha{}^{\beta'} + \quad (5.42)$$

$$\frac{\partial^2}{\partial y^\alpha \partial y^\alpha} \frac{(-1)^{[s]+1} (N_-)^{[s]+1} (\bar{N}_-)^{[s]}}{([s]+1)! [s]} \mathcal{J}_{+\frac{1}{2}}^{[s]+1, [s]}(y^\pm, \bar{y}^\pm|x),$$

$$D_{\mu\beta'}^L \omega^{[s]-1, [s]\mu}_{\beta'} = \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{[s]-2, [s]+1\alpha}_{\beta'} + \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{[s], [s]-1\alpha}_{\beta'},$$

$$D_{\mu\beta'}^L \omega^{[s], [s]-1\mu}_{\beta'} = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{[s]+1, [s]-2\alpha}_{\beta'} + \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{[s]-1, [s]\alpha}_{\beta'} + \quad (5.43)$$

$$\frac{\partial^2}{\partial \bar{y}^{\beta'} \partial \bar{y}^{\beta'}} \frac{(-1)^{[s]+1} (N_-)^{[s]} (\bar{N}_-)^{[s]+1}}{([s]+1)! [s]} \bar{\mathcal{J}}_{-\frac{1}{2}}^{[s], [s]+1}(y^\pm, \bar{y}^\pm|x),$$

$$D_{\alpha\mu'}^L \omega^{[s], [s]-1}_\alpha{}^{\mu'} = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{[s]+1, [s]-2}_\alpha{}^{\beta'} + \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{[s]-1, [s]}_\alpha{}^{\beta'}.$$

Substitution of bilinear J, \bar{J} (5.1) gives

$$D_{\alpha\mu'}^L \omega^{[s]-1, [s]}_\alpha{}^{\mu'} = \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{[s]-2, [s]+1}_\alpha{}^{\beta'} + \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{[s], [s]-1}_\alpha{}^{\beta'} +$$

$$+ \frac{\partial^2}{\partial y^\alpha \partial y^\alpha} \frac{(-1)^{[s]+1} (N_-)^{[s]+1} (\bar{N}_-)^{[s]}}{([s]+1)! [s]} \sum_{p=0}^{[s]} \left(\mathcal{C}_+^{p+1, 0}(y, 0|x) \bar{\mathcal{C}}_-^{0, p}(0, \bar{y}|x) + \bar{\mathcal{C}}_+^{0, p}(0, \bar{y}|x) \mathcal{C}_-^{p+1, 0}(y, 0|x) \right),$$

$$D_{\mu\beta'}^L \omega^{[s], [s]-1\mu}_{\beta'} = \lambda \bar{y}_{\beta'} \frac{\partial}{\partial y^\alpha} \omega^{[s]+1, [s]-2\alpha}_{\beta'} + \lambda y_\alpha \frac{\partial}{\partial \bar{y}^{\beta'}} \omega^{[s]-1, [s]\alpha}_{\beta'} +$$

$$+ \frac{\partial^2}{\partial \bar{y}^{\beta'} \partial \bar{y}^{\beta'}} \frac{(-1)^{[s]+1} (N_-)^{[s]} (\bar{N}_-)^{[s]+1}}{([s]+1)! [s]} \sum_{p=0}^{[s]} \left(\mathcal{C}_+^{p, 0}(y, 0|x) \bar{\mathcal{C}}_-^{0, p+1}(0, \bar{y}|x) + \bar{\mathcal{C}}_+^{0, p+1}(0, \bar{y}|x) \mathcal{C}_-^{p, 0}(y, 0|x) \right).$$

Projecting out the terms, that contain the extra fields $\omega^{[s]-2,[s]+1}$ and $\omega^{[s]+1,[s]-21}$ by the contraction of free indices in with $y^\alpha y^\alpha$ and $\bar{y}^{\beta'} \bar{y}^{\beta'}$, respectively, we obtain the Fang-Fronsdal field equations [25] in AdS_4 with the conformal currents on the right-hand-sides.

6 Conclusion

In this paper the unfolded equations for free massless fields of all spins are extended to current interactions. Interestingly, the resulting equations have linear form, where the currents are realized as rank-two linear fields of [1]. More precisely, the construction of [1] deals with conformal currents built from $4d$ massless fields. Hence, the construction proposed in this paper only describes interactions of massless fields with conformal currents. We have checked in detail how it reproduces usual current interactions for lower spins as well as their generalization to HS sector. Remarkably, the same system reproduces Yukawa interactions in the sector of spins 0 and $1/2$.

More precisely, the set of currents, that results from the construction of [8], is infinitely degenerate with most of the currents being exact, describing no charge conservation. However, the infinite set of currents of a given spin contains one member, that involves a minimal number of derivatives of the constituent fields, and is not exact. In this respect the set of currents resulting from our construction is analogous to that considered recently for the case of any dimension in [31] which is also infinitely degenerate (note, however, that our construction contains HS currents built from fields of different integer and half-integer spins, while in the paper [31] only the HS currents built from a scalar field were considered). Let us stress that exact currents may also play a nontrivial role in the interaction theory: the difference is that nontrivial currents (elements of current cohomology) describe minimal HS interactions while the exact currents (also known as improvements) describe non-minimal HS interactions of anomalous magnetic moment type, that however may also be important in the full interacting HS theory.

The analysis of this paper is performed in the AdS_4 background. The unfolded machinery makes is technically as simple as that in Minkowski case. This should be compared to other approaches to the analysis of HS conserved currents in AdS background [32, 33, 34, 35]. (Note that the case of AdS_3 was considered in [36, 37]).

An interesting problem for the future is to see how the results of this paper are reproduced by the full nonlinear system of equations of motion which is known for HS fields both in AdS_4 [27] and in AdS_d [28] (see also reviews [21, 23]). This may help to reach better understanding of the full nonlinear problem allowing to interpret interactions as the linear problem that involves fields that can either be interpreted as free fields in higher dimensions or as currents in AdS_4 . It should be noted however that to proceed along this direction it is necessary to extend our results to the case of non-gauge invariant HS currents, that are built from HS gauge connection one-forms rather than from the gauge invariant generalized Weyl zero-forms like the generalized Bell-Robinson tensors of [10]. The complication is that currents of this type, like, e.g., the stress tensor built from HS gauge fields, are not gauge invariant as was pointed out in [38]. In fact, it is this property that leads to peculiarities of the HS

interactions [39], that require additional interactions with higher derivatives and non-zero cosmological constant to restore the gauge invariance [26]. It would be interesting to see how this works within the approach presented in this paper.

One of the conclusions of this paper is that, within the unfolded dynamics approach, at least some of interactions can be interpreted in terms of free fields in higher dimensions. The remarkable feature of the unfolded approach is that it makes it easy to put on the same footing field theories in different dimensions. The only source of nonlinearity comes from the realization of higher-dimensional fields as bilinears of the lower-dimensional ones as in Eq. (3.10). It is tempting to elaborate further the interpretation of the obtained results in the context of *AdS/CFT* correspondence. Moreover, we believe that the further analysis of HS gauge theories within the unfolded approach may help to understand the origin of the remarkable interplay between space-times of different dimensions suggested by *AdS/CFT* correspondence [2, 3, 4] going beyond the standard *AdS/CFT* interpretations of HS theories [40, 41, 42, 43, 44, 45, 46, 47, 48]. The results of this paper indicate that HS theories, that involve infinite towers of massless fields associated with infinite dimensional HS symmetries, suggest that the usual space-time picture we are used to work with results from localization of an infinite-dimensional space by virtue of chosen dynamical systems as discussed in [7]. Also we interpret the results of this paper as a further evidence in favor of the idea of an infinite chain of dualities that relate the spaces \mathcal{M}_M with different M , as suggested in [5].

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Appendix A. Weyl sector gluing operators

Consistency conditions of Eq. (4.1) impose restrictions on the gluing operators F^\pm derived below. We begin with the simpler case of flat Minkowski space and then show that the obtained solution also works in *AdS*₄.

Minkowski case

Following Subsection 2.3, we consider the flat limit of equations (4.1), namely (3.11)

$$D_{fl}^{tw}C + e^{\alpha\alpha'} y_\alpha F^+ \frac{\partial}{\partial \bar{y}^{+\alpha'}} \bar{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0} + e^{\alpha\alpha'} y_\alpha F^- \frac{\partial}{\partial \bar{y}^{-\alpha'}} \bar{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0} = 0, \quad (\text{A.1})$$

where

$$D_{fl}^{tw} = D^L + e^{\gamma\beta'} \frac{\partial^2}{\partial y^\gamma \partial \bar{y}^{\beta'}},$$

the rank-two fields $J(y^\pm, \bar{y}^\pm) = \bar{J}(y^\pm, \bar{y}^\pm)$ satisfy flat limit current equations

$$D^L J = e^{\gamma\beta'} \left(\frac{\partial^2}{\partial y^{+\gamma} \partial \bar{y}^{-\beta'}} + \frac{\partial^2}{\partial y^{-\gamma} \partial \bar{y}^{+\beta'}} \right) J \quad (\text{A.2})$$

and $F^\pm(N_\pm, \bar{N}_\pm)$ are unknown “gluing” operators. The consistency conditions of (A.1) require

$$e^{\mu\mu'} e^{\alpha\beta'} \left(\frac{\partial^2}{\partial y^\mu \partial \bar{y}^{\mu'}} (y_\alpha F^A \frac{\partial}{\partial \bar{y}^{A\beta'}} \bar{J}) + y_\alpha F^A \frac{\partial}{\partial \bar{y}^{A\beta'}} D^L \bar{J} \right) \Big|_{y^\pm = \bar{y}^\pm = 0} = 0. \quad (\text{A.3})$$

Taking into account that by (4.4) the operators N_B and \bar{N}_A commute and

$$\left[\frac{\partial}{\partial y^\mu}, F^A \right] = \frac{\partial}{\partial N_B} F^A \frac{\partial}{\partial y^{B\mu}}, \quad \left[\frac{\partial}{\partial \bar{y}^{\mu'}}, F^A \right] = \frac{\partial}{\partial \bar{N}_B} F^A \frac{\partial}{\partial \bar{y}^{B\mu'}},$$

we obtain

$$\begin{aligned} & \left(H^{\mu\alpha} \varepsilon^{\mu'\beta'} + H^{\mu'\beta'} \varepsilon^{\mu\alpha} \right) \left(\frac{\partial}{\partial \bar{y}^{\mu'}} \left(\varepsilon_{\mu\alpha} F^A + y_\alpha \left\{ \frac{\partial}{\partial y^\mu} F^A \right\} + y_\alpha F^A \frac{\partial}{\partial y^\mu} \right) + \right. \\ & \left. y_\alpha F^A \frac{\partial}{\partial \bar{y}^{A\beta'}} \left\{ \frac{\partial^2}{\partial y^{+\mu} \partial \bar{y}^{-\mu'}} + \frac{\partial^2}{\partial y^{-\mu} \partial \bar{y}^{+\mu'}} \right\} \right) \bar{J} \Big|_{y^\pm = \bar{y}^\pm = 0} = 0, \end{aligned} \quad (\text{A.4})$$

which implies

$$H^{\mu'\beta'} \left(\frac{\partial}{\partial \bar{y}^{\mu'}} \left\{ 2 + N_K \frac{\partial}{\partial N_K} \right\} F^A + N_+ F^A \frac{\partial}{\partial \bar{y}^{-\mu'}} + N_- F^A \frac{\partial}{\partial \bar{y}^{+\mu'}} \right) \frac{\partial}{\partial \bar{y}^{A\beta'}} \bar{J} = 0, \quad (\text{A.5})$$

$$H^{\mu\alpha} y_\alpha \frac{\partial}{\partial \bar{y}^A_{\mu'}} \left(\frac{\partial}{\partial \bar{y}^{\mu'}} \frac{\partial}{\partial y^\mu} (F^A \bar{J}) + F^A \left\{ \frac{\partial^2}{\partial y^{+\mu} \partial \bar{y}^{-\mu'}} + \frac{\partial^2}{\partial y^{-\mu} \partial \bar{y}^{+\mu'}} \right\} \bar{J} \right) \Big|_{y^\pm = \bar{y}^\pm = 0} = 0, \quad (\text{A.6})$$

or, equivalently,

$$\left(\left\{ 2 + N_K \frac{\partial}{\partial N_K} \right\} \frac{\partial F^A}{\partial \bar{N}_B} \frac{\partial}{\partial \bar{y}^{B\mu'}} + N_+ F^A \frac{\partial}{\partial \bar{y}^{-\mu'}} + N_- F^A \frac{\partial}{\partial \bar{y}^{+\mu'}} \right) \frac{\partial}{\partial \bar{y}^{A\beta'}} \bar{J} \Big|_{y^\pm = \bar{y}^\pm = 0} = 0, \quad (\text{A.7})$$

$$\left(\frac{\partial^2 F^A}{\partial N_K \partial \bar{N}_B} \frac{\partial^2}{\partial y^K_\mu \partial \bar{y}^{B\mu'}} + F^A \left\{ \frac{\partial^2}{\partial y^{+\mu} \partial \bar{y}^{-\mu'}} + \frac{\partial^2}{\partial y^{-\mu} \partial \bar{y}^{+\mu'}} \right\} \right) \frac{\partial}{\partial \bar{y}^A_{\mu'}} \bar{J} \Big|_{y^\pm = \bar{y}^\pm = 0} = 0. \quad (\text{A.8})$$

This gives the following conditions on F^\pm

$$\left\{ \begin{array}{l} \left\{ 2 + N_K \frac{\partial}{\partial N_K} \right\} \frac{\partial F^+}{\partial \bar{N}_+} + N_- F^+ = 0, \\ \left\{ 2 + N_K \frac{\partial}{\partial N_K} \right\} \frac{\partial F^-}{\partial \bar{N}_-} + N_+ F^- = 0, \\ \left\{ 2 + N_K \frac{\partial}{\partial N_K} \right\} \left(\frac{\partial F^+}{\partial \bar{N}_-} + \frac{\partial F^-}{\partial \bar{N}_+} \right) + N_- F^- + N_+ F^+ = 0, \\ F^+ - \frac{\partial^2 F^-}{\partial N_+ \partial \bar{N}_+} + \frac{\partial^2 F^+}{\partial N_+ \partial \bar{N}_-} = 0, \\ F^- + \frac{\partial^2 F^-}{\partial N_- \partial \bar{N}_+} - \frac{\partial^2 F^+}{\partial N_- \partial \bar{N}_-} = 0. \end{array} \right. \quad (\text{A.9})$$

Note that the last two equations are equivalent to the following useful system

$$\left\{ \begin{array}{l} \frac{\partial}{\partial N_-} F^+ + \frac{\partial}{\partial N_+} F^- = 0, \\ \left(\frac{\partial^2}{\partial N_+ \partial \bar{N}_-} + \frac{\partial^2}{\partial N_- \partial \bar{N}_+} \right) F^+ + F^+ = 0, \\ \left(\frac{\partial^2}{\partial N_- \partial \bar{N}_+} + \frac{\partial^2}{\partial N_+ \partial \bar{N}_-} \right) F^- + F^- = 0. \end{array} \right. \quad (\text{A.10})$$

An elementary straightforward calculation shows that, for arbitrary $n_+, n_- \geq -1$, the operators F^\pm

$$F^\pm = \frac{\partial}{\partial N_\pm} \left((N_+)^{n_++1} (N_-)^{n_-+1} \mathcal{F} \right), \quad (\text{A.11})$$

where

$$\mathcal{F} = \sum_{\bar{n}_-, \bar{n}_+ \geq 0} (\bar{N}_+ N_-)^{\bar{n}_+} (\bar{N}_- N_+)^{\bar{n}_-} \frac{(-1)^{\bar{n}_+ + \bar{n}_-}}{\bar{n}_+! \bar{n}_-! (\bar{n}_+ + \bar{n}_- + n_+ + n_- + 3)!},$$

satisfy Eq. (A.9). Any linear combination of the operators (A.11) also solves Eq. (A.9). The complex conjugated deformation is constructed analogously.

AdS

The deformation to the case of AdS_4 does not require a deformation of the gluing coefficients which remain the same as in Minkowski case.

Indeed, the consistency conditions of (4.1) require

$$\begin{aligned} & \left(H^{\mu\alpha} \varepsilon^{\mu'\beta'} + H^{\mu'\beta'} \varepsilon^{\mu\alpha} \right) \left\{ \frac{\partial^2}{\partial y^\mu \partial \bar{y}^{\mu'}} y_\alpha F^A \frac{\partial}{\partial \bar{y}^{A\beta'}} + y_\alpha F^A \frac{\partial}{\partial \bar{y}^{A\beta'}} \left(\frac{\partial^2}{\partial y^{+\mu} \partial \bar{y}^{-\mu'}} + \frac{\partial^2}{\partial y^{-\mu} \partial \bar{y}^{+\mu'}} \right) + \right. \\ & \left. + y_\mu \bar{y}_{\mu'} y_\alpha F^A \frac{\partial}{\partial \bar{y}^{A\beta'}} + F^A y_\alpha \frac{\partial}{\partial \bar{y}^{A\beta'}} \left(y^+{}_\mu \bar{y}^-{}_{\mu'} + y^-{}_\mu \bar{y}^+{}_{\mu'} \right) \right\} \bar{\mathcal{J}}(y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0} = 0. \end{aligned}$$

Proceeding analogously to the Minkowski case, one can see that (A.12) is true provided that $F^\pm(N_\pm, \bar{N}_\pm)$ satisfy the conditions (A.9) and

$$\frac{\partial}{\partial N_+} F^- + \frac{\partial}{\partial N_-} F^+ = 0, \quad \bar{N}_A \left(1 + \frac{\partial}{\partial \bar{N}_-} \frac{\partial}{\partial N_+} + \frac{\partial}{\partial \bar{N}_+} \frac{\partial}{\partial N_-} \right) F^A = 0. \quad (\text{A.12})$$

However, these additional conditions (A.12) are satisfied by virtue of (A.10). Hence, the solution (A.11) respects consistency in the case of AdS_4 as well.

Appendix B. One-form sector gluing operators, $s \geq 2$

Consistency of (4.7) and (4.1) implies

$$\begin{aligned} & D^{ad} \left(H^{\alpha\beta} \frac{\partial}{\partial y^\alpha} \frac{\partial}{\partial y^\beta} G \mathcal{J} \Big|_{y^\pm = \bar{y}^\pm = 0} \right) + D^{ad} \left(\bar{H}^{\alpha'\beta'} \frac{\partial}{\partial \bar{y}^{\alpha'}} \frac{\partial}{\partial \bar{y}^{\beta'}} \bar{G} \bar{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0} \right) = \\ & = -H^{\alpha\beta} e^{\mu\mu'} \frac{\partial}{\partial y^\alpha} \frac{\partial}{\partial y^\beta} y_\mu F^A \frac{\partial}{\partial \bar{y}^{A\mu'}} \bar{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0 = \bar{y}} - \bar{H}^{\alpha'\beta'} e^{\mu\mu'} \frac{\partial}{\partial \bar{y}^{\alpha'}} \frac{\partial}{\partial \bar{y}^{\beta'}} \bar{y}_{\mu'} \bar{F}^A \frac{\partial}{\partial y^{A\mu}} J \Big|_{y^\pm = \bar{y}^\pm = 0 = y}, \end{aligned} \quad (\text{B.1})$$

where \mathcal{J} , $\bar{\mathcal{J}}$, $\bar{\mathcal{J}}$, J are supposed to satisfy the current equations (3.11). By virtue of (5.12) Eq. (B.1) can be rewritten in the form

$$\begin{aligned} & \mathcal{H}^{\beta\mu'} \frac{\partial}{\partial y^\beta} F^A \frac{\partial}{\partial \bar{y}^{A\mu'}} \bar{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0 = \bar{y}} - \mathcal{H}^{\mu\beta'} \frac{\partial}{\partial \bar{y}^{\beta'}} \bar{F}^A \frac{\partial}{\partial y^{A\mu}} J \Big|_{y^\pm = \bar{y}^\pm = 0 = y} = \\ & = -\mathcal{H}^{\alpha\beta'} D^{L\gamma}{}_{\beta'} \partial_\gamma \partial_\alpha G \mathcal{J} \Big|_{y^\pm = \bar{y}^\pm = 0} + \lambda \mathcal{H}^{\alpha\beta'} y^\gamma \partial_\gamma \partial_\alpha \partial_{\beta'} G \mathcal{J} \Big|_{y^\pm = \bar{y}^\pm = 0} + \\ & + \mathcal{H}^{\alpha\beta'} D^L{}_{\alpha}{}^{\gamma'} \partial_{\gamma'} \partial_{\beta'} \bar{G} \bar{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0} - \lambda \mathcal{H}^{\alpha\beta'} \bar{y}^{\gamma'} \partial_{\gamma'} \partial_\alpha \partial_{\beta'} \bar{G} \bar{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0}. \end{aligned} \quad (\text{B.2})$$

To have a nonzero left hand side in (B.2) let F^\pm be of the form (4.2) with $n_+ = -1$ and $n_- = 2s - 1$, i.e.,

$$F^+ = A(N_-)^{2s} \bar{N}_- \sum_{\bar{n}_+, \bar{n}_- \geq 0} (\bar{N}_+ N_-)^{\bar{n}_+} (\bar{N}_- N_+)^{\bar{n}_-} \frac{(-1)^{\bar{n}_- + \bar{n}_+}}{\bar{n}_-! \bar{n}_+! (\bar{n}_- + \bar{n}_+ + 2s + 2)!}, \quad (\text{B.3})$$

$$F^- = A(N_-)^{2s-1} \sum_{\bar{n}_+, \bar{n}_- \geq 0} (\bar{N}_+ N_-)^{\bar{n}_+} (\bar{N}_- N_+)^{\bar{n}_-} \frac{(-1)^{\bar{n}_- + \bar{n}_+} (\bar{n}_+ + 2s)}{\bar{n}_-! \bar{n}_+! (\bar{n}_- + \bar{n}_+ + 2s + 1)!}. \quad (\text{B.4})$$

Let p be some integer number within the interval $2 \leq p \leq s-2$. Substituting

$$G_{p-s} = \sum_{k=2}^p b_k (N_-)^k (\overline{N}_-)^{2s-k}, \quad \overline{G}_{p-s} = \sum_{k=2}^{(2s-p)} \bar{b}_k (N_-)^{2s-k} (\overline{N}_-)^k \quad (\text{B.5})$$

into (B.2) and using (3.11) along with (B.3) and (B.4), in accordance with the decomposition (4.11) we have

$$\begin{aligned} & \frac{1}{(2s-2)!} \mathcal{H}^{\alpha\beta'} \partial_{-\alpha} \bar{\partial}_{-\beta'} \left(A(N_-)^{2s-2} \overline{\mathcal{J}}^{2s-1,1} - \overline{A}(\overline{N}_-)^{2s-2} J^{1,2s-1} \right) = \quad (\text{B.6}) \\ & = -\lambda \mathcal{H}^{\alpha\beta'} \sum_{k=2}^p b_k k(k-1) (N_-)^{k-2} (\overline{N}_-)^{2s-k} \partial_{-\gamma} \partial_+{}^\gamma \partial_{-\alpha} \bar{\partial}_{-\beta'} \mathcal{J}^{k+1,2s-k+1} + \\ & \quad -\lambda \mathcal{H}^{\alpha\beta'} \sum_{k=2}^{(2s-p)} b_k k(2s-k) (N_-)^{k-1} (\overline{N}_-)^{2s-k-1} (\partial_{-\alpha} \partial_{-\beta'}) \mathcal{J}^{k,2s-k} + \\ & \quad +\lambda \mathcal{H}^{\alpha\beta'} \sum_{n=2}^p \bar{b}_n n(n-1) (\overline{N}_-)^{n-2} (N_-)^{2s-n} \bar{\partial}_{-\gamma'} \bar{\partial}_+{}^{\gamma'} \partial_{-\alpha} \bar{\partial}_{-\beta'} \overline{\mathcal{J}}^{2s-n+1,n+1} + \\ & \quad +\lambda \mathcal{H}^{\alpha\beta'} \sum_{n=2}^{(2s-p)} \bar{b}_n n(2s-n) (\overline{N}_-)^{n-1} (N_-)^{2s-n-1} \partial_{-\alpha} \bar{\partial}_{-\beta'} \overline{\mathcal{J}}^{2s-n,n}. \end{aligned}$$

Substituting

$$b_k = b_{p-s} \frac{(-1)^k}{(2s-k)!k!}, \quad \bar{b}_n = \bar{b}_{p-s} \frac{(-1)^n}{(2s-n)!n!},$$

to (B.6) we have

$$\begin{aligned} & \mathcal{H}^{\alpha\beta'} \partial_{-\alpha} \bar{\partial}_{-\beta'} \left(A \frac{(N_-)^{2s-2}}{(2s-2)!} \overline{\mathcal{J}}^{2s-1,1} - \overline{A} \frac{(\overline{N}_-)^{2s-2}}{(2s-2)!} J^{1,2s-1} \right) = \quad (\text{B.7}) \\ & = +\lambda \mathcal{H}^{\alpha\beta'} \sum_{k=2}^{p-1} b_{p-s} \frac{(-1)^k (N_-)^{k-1} (\overline{N}_-)^{2s-k-1}}{(2s-k-1)!(k-1)!} \partial_{-\alpha} \bar{\partial}_{-\beta'} \left\{ (\partial_{-\gamma} \partial_+{}^\gamma) \mathcal{J}^{k+2,2s-k} - \mathcal{J}^{k,2s-k} \right\} + \\ & \quad -\lambda \mathcal{H}^{\alpha\beta'} \frac{(-1)^p (N_-)^{p-1} (\overline{N}_-)^{2s-p-1}}{(2s-p-1)!(p-1)!} \partial_{-\alpha} \bar{\partial}_{-\beta'} \left\{ b_{p-s} \mathcal{J}^{p,2s-p} - \bar{b}_{p-s} \overline{\mathcal{J}}^{p,2s-p} \right\} + \\ & +\lambda \mathcal{H}^{\alpha\beta'} \sum_{n=2}^{(2s-p-1)} \bar{b}_{p-s} \frac{(-1)^n (\overline{N}_-)^{n-1} (N_-)^{2s-n-1}}{(2s-n-1)!(n-1)!} \partial_{-\alpha} \bar{\partial}_{-\beta'} \left\{ \overline{\mathcal{J}}^{2s-n,n} - (\bar{\partial}_{-\gamma'} \bar{\partial}_+{}^{\gamma'}) \overline{\mathcal{J}}^{2s-n,n+2} \right\} + \\ & \quad -\lambda b_{p-s} \mathcal{H}^{\alpha\beta'} \frac{(\overline{N}_-)^{2s-2}}{(2s-2)!} \partial_{-\alpha} \bar{\partial}_{-\beta'} (\partial_{-\gamma} \partial_+{}^\gamma) \mathcal{J}^{3,2s-1} + \\ & \quad +\lambda \bar{b}_{p-s} \mathcal{H}^{\alpha\beta'} \frac{(N_-)^{2s-2}}{(2s-2)!} \partial_{-\alpha} \bar{\partial}_{-\beta'} (\bar{\partial}_{-\gamma'} \bar{\partial}_+{}^{\gamma'}) \overline{\mathcal{J}}^{2s-1,3}. \end{aligned}$$

To solve (B.7) choosing any solution $\tilde{\mathcal{J}}_{p-s}(y^\pm, \bar{y}^\pm|x)$ of current equations (3.11), we introduce \mathcal{J} and $\bar{\mathcal{J}}$ as follows

$$\mathcal{J} = \bar{b}_{p-s} \sum_{k=0}^{p-2} \left(f_-\right)^k \tilde{\mathcal{J}}_{p-s}, \quad \bar{\mathcal{J}} = b_{p-s} \sum_{k=0}^{2s-p-2} \left(-f_+\right)^k \tilde{\mathcal{J}}_{p-s}, \quad (\text{B.8})$$

where f_\pm are generators (3.17) of sl_2 (3.17).

Now it is easily to see that substituting (B.8) to (B.7) we have

$$\begin{aligned} & \frac{1}{(2s-2)!} \mathcal{H}^{\alpha\beta'} \partial_{-\alpha} \bar{\partial}_{-\beta'} \left(A(N_-)^{2s-2} \bar{\mathcal{J}}^{2s-1,1} - \bar{A}(\bar{N}_-)^{2s-2} J^{1,2s-1} \right) = \\ & = -\lambda(-1)^p \frac{1}{(2s-2)!} |b_{p-s}|^2 \mathcal{H}^{\alpha\beta'} (\bar{N}_-)^{2s-2} \partial_{-\alpha} \bar{\partial}_{-\beta'} \left(f_-\right)^{p-1} \tilde{\mathcal{J}}_{p-s}^{3,2s-1} + \\ & + \lambda(-1)^p \frac{1}{(2s-2)!} |b_{p-s}|^2 \mathcal{H}^{\alpha\beta'} (N_-)^{2s-2} \partial_{-\alpha} \bar{\partial}_{-\beta'} \left(-f_+\right)^{2s-p-1} \tilde{\mathcal{J}}_{p-s}^{2s-1,3}. \end{aligned} \quad (\text{B.9})$$

Hence it immediately follows that

$$\begin{aligned} \bar{\mathcal{J}} &= \lambda |b_{p-s}|^2 (-1)^p A^{-1} \left(-f_+\right)^{2s-p-1} \tilde{\mathcal{J}}_{p-s}, \\ J &= \lambda |b_{p-s}|^2 (-1)^p \bar{A}^{-1} \left(f_-\right)^{p-1} \tilde{\mathcal{J}}_{p-s} \end{aligned} \quad (\text{B.10})$$

solve (B.9). By construction both $\bar{\mathcal{J}}$ and J solve the rank-two current equations (3.11).

Since \mathcal{J} , $\bar{\mathcal{J}}$ of the form (B.8) and J , \bar{J} of the form (B.10) solve (B.7), they obey the consistency conditions (4.7). Note, that to satisfy the reality condition it is necessary to add the complex conjugated sector to the deformation.

Resulting gluing operators with $b_{p-s} = 1 = \bar{b}_{p-s}$

$$G_{p-s} = \sum_{k=2}^p \frac{(-1)^k (N_-)^k (\bar{N}_-)^{2s-k}}{(2s-k)!k!}, \quad \bar{G}_{p-s} = \sum_{n=2}^{(2s-p)} \frac{(-1)^n (N_-)^{2s-n} (\bar{N}_-)^n}{(2s-n)!n!}. \quad (\text{B.11})$$

Resulting deformation

$$H^{\alpha\beta} \frac{\partial}{\partial y^\alpha} \frac{\partial}{\partial y^\beta} \sum_{k=2}^p \frac{(-1)^k}{(2s-k)!k!} (N_-)^k (\bar{N}_-)^{2s-k} \left(f_-\right)^{(p-k)} \tilde{\mathcal{J}}_{p-s}, \quad |_{y^\pm = \bar{y}^\pm = 0} \quad (\text{B.12})$$

$$\bar{H}^{\alpha'\beta'} \frac{\partial}{\partial \bar{y}^{\alpha'}} \frac{\partial}{\partial \bar{y}^{\beta'}} \sum_{n=2}^{(2s-p)} \frac{(-1)^n}{(2s-n)!n!} (N_-)^{2s-n} (\bar{N}_-)^n \left(-f_+\right)^{(2s-p-k)} \tilde{\mathcal{J}}_{p-s} |_{y^\pm = \bar{y}^\pm = 0}, \quad (\text{B.13})$$

Explicit formulas for the important cases are introduced just below. Firstly, let s be integer and $p = s$, then

$$\mathcal{J} = \sum_{k=0}^{s-2} \left(f_-\right)^k \tilde{\mathcal{J}}_0, \quad \bar{\mathcal{J}} = \sum_{k=0}^{s-2} \left(-f_+\right)^k \tilde{\mathcal{J}}_0,$$

$$\bar{\mathcal{J}} = \lambda(-1)^s A^{-1} \left(-f_+ \right)^{2s-p-1} \tilde{\mathcal{J}}_0, \quad J = \lambda(-1)^s \bar{A}^{-1} \left(f_- \right)^{s-1} \tilde{\mathcal{J}}_0.$$

The reality condition requires $\tilde{\mathcal{J}}_0 = \overline{\tilde{\mathcal{J}}_0}$.

Now consider a half-integer s . Choosing $p = [s]$, we have

$$\begin{aligned} \mathcal{J}_{-\frac{1}{2}} &= \sum_{k=0}^{[s]-2} \left(f_- \right)^k \tilde{\mathcal{J}}_{-\frac{1}{2}}, & \bar{\mathcal{J}}_{-\frac{1}{2}} &= \sum_{k=0}^{[s]-1} \left(-f_+ \right)^k \tilde{\mathcal{J}}_{-\frac{1}{2}}, \\ J_{-\frac{1}{2}} &= \lambda \bar{A}^{-1} (-1)^{[s]} \left(f_- \right)^{[s]-1} \tilde{\mathcal{J}}_{-\frac{1}{2}}, & \bar{J}_{-\frac{1}{2}} &= \lambda A^{-1} (-1)^{[s]} \left(-f_+ \right)^{[s]} \tilde{\mathcal{J}}_{-\frac{1}{2}}, \end{aligned}$$

while choosing $p = [s] + 1$ we have

$$\begin{aligned} \mathcal{J}_{+\frac{1}{2}} &= \sum_{k=0}^{[s]-1} \left(f_- \right)^k \tilde{\mathcal{J}}_{+\frac{1}{2}}, & \bar{\mathcal{J}}_{+\frac{1}{2}} &= \sum_{k=0}^{[s]-2} \left(-f_+ \right)^k \tilde{\mathcal{J}}_{+\frac{1}{2}}, \\ J_{+\frac{1}{2}} &= \lambda \bar{A}^{-1} (-1)^{[s]+1} \left(f_- \right)^{[s]} \tilde{\mathcal{J}}_{+\frac{1}{2}}, & \bar{J}_{+\frac{1}{2}} &= \lambda A^{-1} (-1)^{[s]+1} \left(-f_+ \right)^{[s]-1} \tilde{\mathcal{J}}_{+\frac{1}{2}}. \end{aligned}$$

To satisfy reality conditions we set $\tilde{\mathcal{J}}_{+\frac{1}{2}} = -\overline{\tilde{\mathcal{J}}_{-\frac{1}{2}}}$ and

$$\mathcal{J} = \mathcal{J}_{+\frac{1}{2}} + \mathcal{J}_{-\frac{1}{2}}, \quad \bar{\mathcal{J}} = \bar{\mathcal{J}}_{+\frac{1}{2}} + \bar{\mathcal{J}}_{-\frac{1}{2}}, \quad \bar{J} = \bar{J}_{+\frac{1}{2}} + \bar{J}_{-\frac{1}{2}}, \quad J = J_{+\frac{1}{2}} + J_{-\frac{1}{2}}. \quad (\text{B.14})$$

Appendix C. Spin 3/2 one-form sector

Here we consider in detail the special case of $s = 3/2$.

Substituting

$$G = \frac{1}{2} b (N_-)^2 \bar{N}_-, \quad \bar{G} = \frac{1}{2} \bar{b} N_- (\bar{N}_-)^2,$$

into the consistency conditions (B.2) and using (3.11) in accordance with the decompositions (4.11), we have along with (B.3) and (B.4)

$$\begin{aligned} & 2\mathcal{H}^{\beta\mu'} \frac{\partial}{\partial y^\beta} F^A \frac{\partial}{\partial \bar{y}^{A\mu'}} \bar{\mathcal{J}}^{2,1} - 2\mathcal{H}^{\mu\beta'} \frac{\partial}{\partial \bar{y}^{\beta'}} \bar{F}^A \frac{\partial}{\partial y^{A\mu}} J^{1,2} = \\ & = b\mathcal{H}^{\alpha\beta'} D^{L\gamma}{}_{\beta'} \partial_{-\gamma} \partial_{-\alpha} \bar{y}^{\nu'} \partial_{-\nu'} \mathcal{J}^{2,1} - b\lambda \mathcal{H}^{\alpha\beta'} y^\gamma \partial_{-\gamma} \partial_{-\alpha} \partial_{-\beta'} \mathcal{J}^{2,1} + \\ & - \bar{b}\mathcal{H}^{\alpha\beta'} D^L{}_{\alpha}{}^{\gamma'} \partial_{-\gamma'} \partial_{-\beta'} y^\nu \partial_{-\nu} \bar{\mathcal{J}}^{1,2} + \bar{b}\lambda \mathcal{H}^{\alpha\beta'} \bar{y}^{\gamma'} \partial_{-\gamma'} \partial_{-\alpha} \partial_{-\beta'} \bar{\mathcal{J}}^{1,2}. \end{aligned}$$

Hence, using Eq. (3.11), we obtain

$$\begin{aligned} \mathcal{H}^{\beta\mu'} \frac{\partial}{\partial y^\beta} F^A \frac{\partial}{\partial \bar{y}^{A\mu'}} \bar{\mathcal{J}}^{2,1} &= -b\lambda \mathcal{H}^{\alpha\beta'} N_- \partial_{-\alpha} \partial_{-\beta'} \mathcal{J}^{2,1} + \bar{b}\lambda \mathcal{H}^{\alpha\beta'} \partial_{-\gamma'} \partial_{-\beta'} N_- \partial_+{}^{\gamma'} \partial_{-\alpha} \bar{\mathcal{J}}^{1,2}, \quad (\text{C.1}) \\ \mathcal{H}^{\mu\beta'} \frac{\partial}{\partial \bar{y}^{\beta'}} \bar{F}^A \frac{\partial}{\partial y^{A\mu}} J^{1,2} &= b\lambda \mathcal{H}^{\alpha\beta'} \partial_{-\gamma} \partial_{-\alpha} \bar{N}_- \partial_{-\beta'} \partial_+{}^\gamma \mathcal{J}^{2,1} - \bar{b}\lambda \mathcal{H}^{\alpha\beta'} \bar{N}_- \partial_{-\alpha} \partial_{-\beta'} \bar{\mathcal{J}}^{1,2}. \end{aligned}$$

Then choosing F^\pm of the form (B.3), (B.4), where $n_+ = -1$ and $n_- = 2$, and $\mathcal{J} = \widetilde{\mathcal{J}}_{-\frac{1}{2}} = -\overline{\widetilde{\mathcal{J}}_{+\frac{1}{2}}}$ in the notations Appendix B, it is easily to prove that

$$\overline{\mathcal{J}} = -\lambda A^{-1} \mathcal{J} + \lambda A^{-1} \left(-f_+ \right) \overline{\mathcal{J}}, \quad J = -\lambda \overline{A} \overline{\mathcal{J}} + \lambda \overline{A}^{-1} \left(f_- \right) \mathcal{J} \quad (\text{C.2})$$

solve (C.1) .

Note, that (C.2) coincides up to a numerical factor with (B.14) under conventions that all the sums of the form $\sum_{k=0}^{-1}(\dots)$ or $\sum_{k=2}^1(\dots)$ are zero.

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